X. *A Critical Study of Spectral Series.—Part V. The Spectra of the Monatomic Gases.*

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[Plates 2–5.]

Contents.

THE present communication has two objects. Its subject matter is an attempt to obtain some knowledge of the series relations in the spectra of the group of the monatomic rare gases Ne to RaEm, whilst the methods employed will serve to illustrate the fundamental importance as instruments for further research of the new facts brought to light in the previous communications.* The importance of the first object will be generally acknowledged, but it does not yet seem to be realised how definite and exact those new relationships are, even in their as yet undeveloped form, and how powerful an instrument is placed in our hands for the analysis of spectra. It may be well therefore to commence by a brief *résumé* of some of these laws as applied in the succeeding pages. Further, as the establishment of the results obtained must by its nature depend on the numerical comparison of a very large number of lines in all five spectra, and as this evidence must be fully set out to

* "A Critical Study of Spectral Series," Part I., 'Phil. Trans.,' A, vol. 210, p. 57.
„ „ „ Part II. „ „ 212, p. 33.
„ „ „ Part III. „ „ 213, p. 323.
„ „ „ Part IV. „ „ 217, p. 361.
These will be referred to respectively as [I.], [II.], [III.], [IV.].

enable a specialist judgment to be formed on it, the communication has unfortunately become very lengthy. The mass of detail will perhaps be rather dreary to the general reader not specially interested in this line of study. It is apt also to hide by its amount and complexity the general conclusions arrived at. I propose therefore to give a slight general survey of these conclusions before giving the evidence.

As is well known the wave-numbers of series lines depend on four types of sequences $p(m)$, $s(m)$, $d(m)$, $f(m)$, and that in any one series they depend on the differences between one sequent of one type and the successive terms of the sequence of another type. These sequences are all of the form $N/\{\phi(m)\}^2$, where N is RYDBERG's constant and $\phi(m)$ is of the form m+fraction, the fraction being, as a rule, determinable as a decimal to six significant figures. Our aim is to discover the properties of these functions. The fractional part depends in some way on the order m, although whether it can be considered a definite function of m in the ordinary sense is doubtful.* This fractional part will be referred to as the mantissa, and in dealing with it, it will be regarded as multiplied by 10^6, *i.e.*, as if the decimal point were removed.

The Oun.—It is found† that in each element a constant quantity particular to each element plays a fundamental part in the constitution of the sequences. This is called the oun. The d and f sequences depend in definite ways on multiples of this quantity, whilst it also enters into the constitution of the p and s. Its determination is therefore for each element a matter of the first importance. Denoting its value by δ_1, the quantity $\delta = 4\delta_1$ is of such frequent recurrence that it is useful to treat it as one datum. The oun is accurately proportional to the square of the atomic weight, and is given by $\delta = (361{\cdot}8 \pm {\cdot}1)(w/100)^2$, where w denotes the atomic weight.

In the case of doublet or triplet series, the corresponding separations between them are due to different limits whose mantissæ differ by amounts Δ or Δ_1, Δ_2 (say). In all cases these are found to be integral multiples of the oun. For triplets $\Delta_1 : \Delta_2$ is always somewhat greater than 2.

In the case of D series where satellites occur, the separations of the latter are due to differences in their d sequences. The mantissæ of these latter again differ by quantities which are multiples of the oun, and in the case of triplets they appear in normal types to be very close to the ratio 5 : 3.

The d Sequence.—In the normal type the sequent of the extreme satellite has its mantissa a multiple of Δ_2. The only known exceptions are found in Sr, Cd which show the multiple law, Sr in d_{12} and Cd in d_{11} instead of in d_{13}. In both these cases also the Zeeman pattern is abnormal. As the main lines D_{11} (and in triplets D_{12} also) have their mantissæ greater than that of the outer satellite by multiples of the oun, it follows that all the d sequences for the first order have mantissæ multiples of the oun. It is probable that this is true for all orders of m, but the data are not

* 'Astro. J.,' 44, p. 229, see also [III., p. 339].

† [III.], also 'Proc. R. S.,' A, 91 (1915).

sufficiently accurate to prove this, although they obey the rule within error limits. [III., p. 340.]

The f Sequence.—This sequent of the first order also has the multiple of Δ_2. The material at disposal is not so comprehensive as in the case of the D series, for, except in the second group of the periodic table, the F lines occur chiefly in the ultra-red. The proof of the above statement is perhaps, therefore, not so conclusive as in the case of the d sequence. It completely stands the test however in the rare gases. There seems some evidence that F series also show a satellite effect in a small degree —of one or two ouns. In the second group it seems to be a general rule that in many of the low orders ($m = 1, 2\ldots$) the f sequents receive very large displacements from their normal value, so that a normal line is much weaker or is altogether absent and replaced by others separated from it by considerable numbers. This also is found to be the rule in the present case.

Displacement.—Regarding the ordinary doubtlet or triplet series we may consider the second (or third) as displaced from the first by the deduction of a certain number of ouns from the mantissa of the limit; or better perhaps regard the last satellite set as the fundamental one and the others as displaced by the addition of ouns. When such displacements occur in the limit of one line the new one is indicated by writing the displacement on the left. Thus $S_2(m) = (-\Delta_1) S_1(m)$ or $S_1(m) = (\Delta_1) S_2(m) = (\Delta_1+\Delta_2) S_3(m)$. With satellites, on the other hand, the similar effect is produced in the sequence terms. In this case it is entered on the right. Thus $D_{12}(m) = D_{11}(m)(-x\delta_1)$ or $D_{11}(m) = D_{12}(m)(x\delta_1)$; $D_{22}(m) = (-\Delta_1) D_{12}(m) = (-\Delta_1) D_{11}(m)(-x\delta_1)$. Displacements of both kinds are very common in spark spectra and put themselves specially in evidence in the succeeding pages. A normal line may not only show lines displaced from it, but often it appears to be replaced by them, and, in general, when it does not disappear its intensity is abnormally low. This is practically what happens in the D satellites. The D_{13}, D_{23}, D_{33} appear to be the normal lines in which we should expect descending order of intensity; but most of the energy (or the majority of the emitting centres) appears carried over to the more intense and displaced set D_{12}, D_{22}; and, again, most of what should be expected in D_{12} is carried over to become the strongest line D_{11}. Frequently the D_{13} line has disappeared and only the fragment D_{23}, D_{33} of the triplet left. In general, the D_{11} lines of any element are the strongest of the series. But in the present vacuum tube spectra (spark type) we shall find very frequently that the line required for D_{11} is comparatively weak, and in this case there appear other lines related to it by oun displacements chiefly in the limit. As the real existence of these displaced series is a matter of some importance considerable space has been given in the discussion of the X spectrum (p. 399) to its demonstration in the case of two series depending on the limits $(\pm 2\delta_1) D(\infty)$. It seems a peculiarity of these displacement series that a term of one series may be absent but appear in another. Thus $(-2\delta_1) D(m)$ may not be observed, but a $(+2\delta_1) D(m)$ may be and *vice versâ*. The presence of a similar effect in the sequent terms of F series has been referred to above. One good illustration of double displacements fully established is found in the KrS series (p. 349), in which the indications are shown for $m = 1, 2, 3$. A knowledge of the laws governing displacements is much to be desired. Very little is known at present.

Linkages.—Arc spectra are distinguished, as a rule, by the presence of well-defined series, depending on single groups of P.S.D.F. type. In spark and vacuum tube spectra, however, these seem to be weakened, and a very large number of other lines appear which are related to one another by certain constant separations (links) to form congeries of linkages each connected to a series line. These links can be calculated when the values of Δ, or of Δ_1, Δ_2, are known. The evidence for these was given in [IV.]. There appear to be links of several types. Those already discussed are of two types: (1) separations between successive double displacements of Δ_1 on either side of $S_2(\infty)$ or $p_2(1)$; (2) displacements of Δ_1 on either side of $P(\infty)$ or $s(1)$. Of these, use is confined almost entirely in the present communication to one only of type (1) and both of type (2). They are

$$e = (-2\Delta_1)\,p_2(1) - (2\Delta_1)\,p_2(1),$$

or

$$= (-3\Delta_1)\,S_1(\infty) - (\Delta_1)\,S_1(\infty),$$

$$u = s(1) - (\Delta_1)\,s(1), \qquad v = (-\Delta_1)\,s(1) - s(1).$$

These links themselves may also be subject to small displacements by having their sources on, say, $(x\delta_1)\,S_1(\infty)$ instead of $S_1(\infty)$. For the present purpose, however, no use can be made of these.

In [IV.] the prevalence of these separations in a spectrum in excess of their occurrence from mere chance was exhibited in a series of curves with abscissæ = separation and ordinates = number of occurrences within a given small amount on each side. Such occurrency curves are also given here for the e links and for the u, v of Kr in Plate 2. The e links seem to be a normal accompaniment to series lines (often displaced, however, when directly attached to those of low order). A further peculiarity of these linkages is the prevalence of the combination $e \pm u$, or $e \pm v$. They are indicated by writing the letter denoting the link to the left of the line when deducted and to the right when added. Thus, in the example below, $44236 = e.47419$, or $47419 = 44236.e$.

Sounding.—In the following pages the unravelling of the complete series of linkages has not been touched upon, but the e, u, v links have been used for testing the existence of lines outside the observed region, a method we may call sounding. A link thus used may be referred to as a sounder. In this way it is possible to obtain evidence of the existence, or of the exact value, of a calculated line which lies beyond the region observed. It may even serve as evidence for the real existence of a line in the observed region too weak to have been observed, for it was shown in [IV.] that the e link appeared to have a tendency to increase the intensity of one of the two lines to which it was attached. The method may be illustrated by an example from KrS. In Kr, $e = 3183{\cdot}35$. The value of $S_1(5)$ as calculated from the formula is 47419·39, which is in the ultra-violet outside the observed region. But $47419{\cdot}39 - e = 44236{\cdot}04$. This is within the observed region, and as a fact the corresponding line is found at 44237·61 with $d\lambda = -{\cdot}08$ if e is free from error. As an individual case this might be due to a coincidence, but when the same effect occurs with line after line the cumulative effect becomes convincing. To see this it is necessary to get at a glance a survey of all the cases, and for this purpose they are exhibited in sets of diagrams in Plate 3. These diagrams also include links within the observed region in order to show that where the method can be tested it holds. It may be specially noted how the similar arrangement of sounders holds for the same order in the three lines of the same triplet, and how in certain cases the u, v seem alternative. *Cf.* for example XS (1, 3, 4, 7, 8), or the main lines of the three parallel D sets in X, viz. $(2\delta_1)D_{11}$, D_{11}, $(-2\delta_1)D_{11}$, or particularly the prevalence of the $-(e+v)$ combinations in the unobserved lines for KrD. In RaEm these links are too large to be of wide application and in Ne too small to be of use. In RaEm the e link is 23678 and can reach from the unobserved ultra-red across to the unobserved ultra-violet. In Ne the e link is 196, so that its reach is too small to be useful. As this method of sounding is new and clearly of importance if substantiated, considerable attention has been given to its illustration, but as the details themselves are only necessary for a critical study they have been printed in smaller type and may be omitted on a first reading.

Abnormal D *triplet Separations.*—It has generally been held since RYDBERG's discovery of the satellite systems that the triplet separations for the D and S series are the same. The actual measures did not absolutely prove this, in fact, they indicated small differences, but the accuracy was not sufficient to establish a real difference especially as against a natural bias to expect equality. MEGGERS,* however, has recently placed it beyond doubt that frequently the separations are really different. In Group I., for instance, the separations as measured from D lines are less than those determined from S lines. In the rare gases also this difference appears quite decisively, but here (group 0) the separations as determined from the D lines are, in general, larger than those from S. The key to the explanation is found in the fact that the difference between the two determinations diminishes with increasing order—in other words, that the sequent in the same set of satellites is not the same, and that in a large number of cases the value of $\nu_1+\nu_2$ is the same in both S and D although ν_1, ν_2 themselves are different. It is found to be completely explained by the displacement of one or two ouns between the sequents of the first or second members of a triplet. Sometimes it occurs in the third member. The same explanation accounts for the fact that the F separations are frequently smaller than the

* 'Bur. Stand.,' Washington, No. 312 (1918).

corresponding observed satellite separations. It also accounts for the appearance of F satellites as shown here and in [III., pp. 389–395] in connection with the F lines in the alkaline earths. The matter is considered in detail for Kr (p. 363) and is found to hold for the cases of the other rare gases.

The Atomic Weight.—It is clear that an accurate knowledge of a first f sequent, or of a d sequent which belongs to the satellite involving the Δ_2 multiple, gives the means of determining the oun to in general a unit in the sixth significant figure. For these mantissæ are usually of the order of magnitude of 0·8 and are known to six figures. Hence, if the multiple is known, Δ_2 can itself be determined, and since Δ_2 is a known multiple of the oun (determined by the S multiplet separations), the oun is also known to the same degree of accuracy. Further, as the sources of determining it are often quite independent they serve as tests of the determinativeness of the oun itself to the same degree of accuracy. When Δ_2 is considerable, its value is known sufficiently well for it to determine the multiple, and then this exact integer conversely gives the exact value of Δ_2. In the cases of A and Ne, however, the values of Δ_2 are too small to determine uniquely this multiple directly. The difficulty, however, is surmounted by obtaining successively values with increasing accuracy from other considerations until the final test can be applied. As a fact the Ne oun is amongst the most accurate found. Its determination (p. 461) is specially interesting, and indeed is only possible because the material at disposal depends on interferential measures and large accurately known separations. That of X also is a good determination, and is interesting as depending on a number of quite independent data.

As the oun is proportional to the square of the atomic weight within the limits of error of determination of the latter, it is natural to assume that the relation is exact and that $\delta = q.w^2$, where q is a number between $361{\cdot}8 \pm {\cdot}1$. If this were sustained it would be possible to obtain w with twice the degree of accuracy of the oun and therefore far in advance of any obtainable by chemical methods. In fact the question is raised as to what is actually understood by the atomic weight. Does it refer to the mass of the positive nucleus, or to that and all or a portion of the electrons? The hope might even be entertained of obtaining by this method some knowledge of the number of electrons partaking in the emission of a line if slight changes in the oun could be found. For instance, we shall find in these spectra not a single group of S, D, or F series as in arc spectra, but several independent groups, viz., d and f sequents, depending on different multiples of Δ_2. If these gave slightly different values of the oun it could be explained by a transference of electrons. There is little evidence of such variation, but it might occur, for instance, in the oun as deduced respectively from Δ_1 and Δ_2. As Δ_1 depends alone on the measurement of the ν_1 separation of a triplet it is not susceptible of such exact determination as Δ_2, and, as a fact, a suspicion sometimes arises that such a slight difference may exist, and that δ from ν_1 is somewhat less than from ν_2 [III., p. 333] as also here.

Suppose the atomic weight is W and the number of electrons involved is xW. Then the oun is given by

$$\delta_x = q\,(\mathrm{W}+x\mathrm{W}/1850)^2 = \delta_0\left(1+\frac{x}{925}\right).$$

If another value depends on y electrons

$$\delta_y = \delta_0\left(1+\frac{y}{925}\right),$$

whence

$$x-y = 925\,\frac{\delta_x-\delta_y}{\delta} = 925\,\frac{d\delta}{\delta},$$

which gives the transference. At present these considerations are only of speculative interest, but a numerical illustration is given below (p. 381) in connection with Kr.

The results obtained in this investigation have given the oun with much greater exactness than any value obtained in [III.], even than that of Ag. The value of $q = \delta/w^2$ has been determined [III., p. 404] as near 361·75 with Ag = 107·88. I now believe from later work that the true value is closer to this than I thought at that time, but in any case it is far less accurate than the ouns themselves. While, therefore, we can use the ouns to give extremely accurate values of the ratios of the atomic weight of the gases, the actual values in terms of Ag are not so exact, although more accurate than those obtained by chemical means. This statement of course depends on the supposition of the exact proportionality of oun and square of atomic weight.

The values of δ as obtained later are here collected and the atomic weight deduced from them by taking q = 361·75.

	Ne.	A.	Kr.	X.	RaEm.
δ . . .	14·4708 ± ·0006;	57·9209 ± ·002;	249·536 ± ·004;	611·0100 ± ·0017;	1787·024 ± ·05
W . . .	20·0005 ± ·0004;	40·0141 ± ·0006;	83·0543 ± 0006;	129·963 ± ·00018;	222·259 ± ·003
Chemical .	20·2	39·88	82·92	130·2	222 to 222·4

It will be seen that in all cases the spectral determinations are much closer to integral values than the chemical, except in the case of RaEm as estimated from HÖNIGSCHMIDT'S value for Ra. In this case, however, the spectral material is defective. It is shown from one of the criteria that a value of the oun = 1785·23 is just possible but improbable, or = 1783·38 almost impossible. These would give respectively w = 222·148 ± and 222·033 ±. It is curious also that from the defective observational work for Ra [III., p. 327] the value of δ from $\nu_1+\nu_2 = 254096 = 137\delta$, whence w = 226·43 is also greater than HÖNIGSCHMIDT'S and more in accordance with the value obtained by Mme. CURIE. The value for the Emanation is, however, much more reliable than the above for Ra. If, regarded as a whole, the deviations from the chemical values (RaEm excepted) are greater than chemists will allow possible, it would seem that in this case we are not dealing with precisely the same entity in the two cases.

Special F *Series*.—There appears to be a remarkably stable triplet series of the F type apparent in most of the gases, but more especially evident in X, in which element it was first noticed. Not only are the lines strong and present in a large number of orders, but they appear, at least in X, to be little susceptible to displacements such as are common in other types. The separations are 1864, 829. The occurrency curve for 1864 is shown in Plate 2, fig. 3. In this, in strong contrast to other such curves, it rises to a very high single peak and is practically symmetrical on both sides of the peak. The similar curve for A is shown in Plate 2, fig. 5.

Summation Series.—In the investigation of this XF series a quite new type of series was brought to light. The hitherto recognised series appear as the differences of two terms A−B. The new one has its wave-numbers of the form A+B. In other words, where the old series are difference frequencies the new ones are the corresponding summation frequencies. The notation adopted is to write the corresponding terms in Clarendon type. Thus

$$\mathrm{F}(m) = \mathrm{A}-f(m), \qquad \mathbf{F}(m) = \mathrm{A}+f(m).$$

The list of the lines in X is given on p. 385 up to $m = 30$. For low orders, $m < 3$, the lines are in the ultra-violet and have to be sounded for. Similar summation series coupled with other F series are also common. It probably explains also the crowding of F separations in spectra like that of Cu in short wave regions far beyond the F limit which has always appealed to me as a difficulty. It is possible that summation series may also exist for the P.S.D. series in all elements, but, as a rule, the limits of these are far larger than the $\mathrm{F}(\infty)$, with the consequence that any **P.S.D.** lines must lie very far in the ultra-violet, a fact which explains why such types if existing have not hitherto been recognised. The existence of these summation series is thoroughly established and their importance as bearing on theories of the origin of spectral lines is evident. They would seem difficult to explain on any of the current theories. But apart from this the existence of the type is of great value for quantitative determinations. This is fully dealt with on p. 384 and it need not be recapitulated here. Its importance for this purpose may be realised when it is seen that it forms the starting point in the analysis of the RaEm spectrum, that it settles in a quite definite way a difficulty arising in the evaluation of the oun in Kr, and that it fixes a very accurate value for the limit of the 1864 series in X, thus simultaneously fixing a particular *d* sequent subject only to observation error in one line.

Groups of D *and* S *Series*.—Not only do we meet with different groups of D series depending on different multiples of Δ_2,* but in the case of Kr there appear to be two

* As an example, see p. 403, in X with groups depending on $70\Delta_2$ and $79\Delta_2$.

sets of lines suitable for S_3—in other words, there are quadruplets. Whilst the two sets S'_3, S_3 give different separations with S_2, and consequently different Δ'_2, Δ_2, they give the same oun, and in connection with them appear two D groups whose outside satellites depend one on a multiple of Δ'_2 and the other on a multiple of Δ_2. It is to be suspected that this is only one example of what may be a common occurrence in spark spectra.

The order of presentation is generally that in which the investigation was taken. The key was found in obtaining the KrS system, a result first rendered possible by the publication of wave-lengths in the ultra-violet by LEWIS (1915). Amongst them the KrS (1) triplet was found. XS, AS come next in order of definiteness. The spectra of RaEm and Ne are more difficult to deal with, the first because of its defectiveness in range, number of lines, and accuracy, and the latter because of the smallness of its oun and its triplet separations. After the S series of Kr, X, A come the D and F series of Kr, X, the spectrum of RaEm, the D and F for A, and, lastly, the whole sets for Ne. Led by possibly a false analogy to He [I., p. 105], in which doublet series appear in the blue spectrum, the blue spectra were chosen for investigation, and the family group being of even order triplets were looked for. In Ne, with a single spectrum of composite character, the results obtained may have some reference to the red type as well as the blue, especially in connection with certain remarkable constant separations found by WATSON and analogous to the RYDBERG constant separations in the red spectrum of A. One is inclined to think that these red spectra consist mainly of lines of the F type. But the red or first spectra are outside the scope of the present communication. Although it is a very lengthy one as it stands only the beginning of an analysis has been made. The aim has been to lay the foundation for the series framework of this family of elements, and little beyond has been done. The linkages, as a whole, have not been isolated, the red spectra not touched upon, and many interesting effects which will require clearing up are passed over without reference. A great field for investigation is open in these and other spectra for any who are willing to give the necessary time and patience. In some few cases the presentation might have been slightly shortened by merely stating the final result and showing how the necessary conditions are satisfied. But not only would this have passed over certain phenomena of special interest, but one of the objects of the present communication would have been missed, viz., to illustrate the power of the new facts to guide a search even when the details are most bewildering. Moreover, the evidence itself is the more striking when developed from step to step than when the result is directly presented as a finished product.

Krypton.—Krypton shows two spectra, without and with capacity, the former in the red region and the latter further towards the blue. We have measures of some of the stronger lines by RUNGE, and a considerable number of weaker lines, not observed by others, by LIVEING and DEWAR, although the latter are only given to the nearest Ångstrom. The most complete and reliable sets of measurements are by BALY* (red spectrum 6456—3502; blue 5871—2418), and LEWIS† (blue 2416—2145), both of about the same degree of accuracy with probable error in the neighbourhood of ·03 A. Of exact measures there are only two by FABRY and BUISSON‡ for lines at 5870.9172, 5570.2908 I.A. In the red spectrum RUNGE has pointed out constant separations of 945, to which PAULSON§ has added three others. The observations of LEWIS gave me the first clue to the KrS set of lines and thus formed the starting point for the present communication, although a great deal of preliminary discussion of material for this group of elements had been previously done, especially in connection with the separations for certain linkages. In the case of Kr a very large number of separations in the neighbourhood of 786 to 788 had been found, connected also with others of 309, indicating groups of triplets having these values for ν_1, ν_2. Amongst LEWIS' lines a set was found with separations in the reverse order, clearly pointing to a first set of $-S(1)$ or $+P(1)$ lines and corresponding sets for other orders were then easily found. It would seem that there are always a considerable number of separations governed by oun displacements in the limits of the first order, and that of these, three seem to be of a more stable value and correspond to normal triplets. For instance, in all these gases we find a very large number of cases where a S_1 or D_1 line is followed by a line with a separation very close to $\frac{1}{2}\nu_1$. They force themselves on attention on account of their value being so close to the half of a number being sought for, and others may be present although they have not been looked for. In the present case two alternative sets of lines for the S_3 series, one with $\nu_2 = 309$ and the other with $\nu_2 = 341$ appear. In the original search the former was taken because it is reproduced in the D series as well. But later certain difficulties in the determination of the oun, combined with the fact that the corresponding oun multiple in Δ_2, although quite definite, is out of step with the march of their values in the other gases, led me to include the second. This gives a multiple quite in step with the others, and also affords the means of obtaining a good approximation to the oun.

The lines are given in the following table, which also embrace a few obtained by sounding, both wave-lengths and wave-numbers are given :—

* 'Trans. Roy. Soc.,' A, vol. 202, p. 183 (1903).
† 'Astro. Journ.,' 43, p. 67 (1915).
‡ 'C.R.,' March 25 (1913).
§ 'Kong. Fys. Sälls. Hand.,' N.F. 25, Nr. 12.

KrS.

	S_1.		S_2.		S'_3.		S_3.
1.	−(10) 2353·95 42468·98	**786·52**	−(10) 2398·38 41682·46	**309·20**	−(3) 2416·31 41373·26	**340·51**	(−1) 2418·13‡ 41341·95
2.	(10) 3778·23 26460·07	**786·45**	(9) 3669·16 27246·62			**341·49**	(4) 3623·74 27588·11
3.	(3) 2489·51 40156·61	**769·80**	(1) 2442·68* 40926·41		‡		‡
4.	(1) 2216·72 45097·86	**774·3**	(1) 2179·3† 45872·2 ± 2	**311·4**	(2) 2164·6 46183·6 ± 2	**786·45 + 339·86**	(2) 2162·7 46224·17 ± 2
5.	(2) 2259·83.*e* (47421·14)					**786·45 + 343·5**	(6) 2299·02.*e.u.* (48551·12)
6.	(4) 2362·18.2*e* ? (48688·12) or (48698·34) §	**788·06**	(1) 2159·5.*e* (49476·18)	**301·54**	(6) 2302·88.2*e* (49777·72)	**786·45 + 340·4**	(8) 2300·35.2*e* (49825·2)

The first three S_1 lines give for the formula

$$n = 51651{\cdot}29 - N/\{m + {\cdot}093630 - {\cdot}014156/m\}^2.$$

The calculated wave-numbers for $m = 4 \ldots 7$ are in order 45095·25, 47419·39, 48695·38, 49470·47. The first gives O−C, $d\lambda = {\cdot}12$; the others are outside the observed region but are reached by sounding and give O−C values of −·08, ·12, ·22, the errors including those of the sounders.

Quantities relating to the separation 309 will be denoted by dashed letters. With the limit $51651{\cdot}29 + \xi$ and separations $786{\cdot}45 + d\nu_1$, $309{\cdot}20 + d\nu'_2$, $341{\cdot}16 + d\nu_2$, || the values of Δ_1, Δ_2 are found to be

$$\Delta_1 = 10969 - {\cdot}316\xi + 13{\cdot}79\, d\nu_1 = 44\,(249{\cdot}30 + {\cdot}314\, d\nu_1 - {\cdot}007\xi),$$
$$\Delta'_2 = 4244 - {\cdot}121\xi + 13{\cdot}67\, d\nu'_2 = 17\,(249{\cdot}63 + {\cdot}80\, d\nu'_2 - {\cdot}007\xi),$$
$$\Delta_2 = 4680 - {\cdot}121\xi + 13{\cdot}65\, d\nu_2 = 18\tfrac{3}{4}\,(249{\cdot}64 + {\cdot}73\, d\nu_2 - {\cdot}007\xi).$$

The value of the oun is thus given by $\delta = 249{\cdot}30 + {\cdot}314\, d\nu_2$, $249{\cdot}63 + {\cdot}8\, d'\nu_2$, with the uncertainty usually found from triplet separations [see III., p. 332]. From the limit $= p(1)$ and Δ_1 the values of the a, b, c, d, e links are at once found. The result for $e = p_2(1)(-2\Delta_1) - p_2(1)(2\Delta_2)$ is 3183·35. Since $S_1(1) = p(1) - s(1)$,

* ? $S_2(3)(-9\delta)$, see displaced sets below.

† Is $F_8(2)$, see p. 376.

‡ In region at very end of BALY'S list; 41341 is his last.

§ (6) 2291·26.*e.u.*

|| Obtained as least square value from m = 1, 2, 3, supposing λ equally probable.

$s(1) = 94120{\cdot}27 + \xi = N/\{1{\cdot}079474 - 5{\cdot}73\xi\}^2$. From this the u, v links, viz., $u = s(1) - s(1)(\Delta_1)$, $v = s(1)(-\Delta_1) - s(1)$ can be calculated. The results are

$$u = 1884{\cdot}03 + 2{\cdot}33\, d\nu_1 - {\cdot}02\xi,$$
$$v = 1942{\cdot}44 + 2{\cdot}48\, d\nu_1 - {\cdot}03\xi,$$

with

$$e = 3183{\cdot}34 + 4{\cdot}096\, d\nu_1 - {\cdot}006\xi.$$

The occurrency curves for these are shown in Plate 2 (fig. 1). It will be seen at once that the maximum occurrences appear with sharp peaks in close agreement with the calculated values.

The calculated lines for $m > 4$ are outside the observation region, but their existence can be indicated by using the above links as sounders. At the same time, in order to give more confidence in the application of the method, the similar linkages are examined for the lines which are observed. It is, of course, understood that the existence of a single link is no conclusive evidence of the existence of the unseen line, as it may be a coincidence. The evidence is cumulative and is seen at a glance in the diagram in Plate 3 (fig. 1). The actual data are given here. In the application of this the frequent link modification must be allowed for, as well as the effect of observation errors. Consequently separations deviating by not more than two or three units from the values determined above are admitted. As a fact, this will exclude a number of real link connections as well as include a certain number of pseudo ones. But by limiting the deviations to these small amounts the conclusions drawn will be more reliable.

Sounding Data.

S (1).

(2) 37340			(1) 39799	**1882·57**	
1944·49					
(1) 39284	**3184·49** S_1		(1*n*) 38497	**3185·23** S_2	S_3
1882·11					
(3) 41166			(1) 39741	**1940·62**	

Here the u, v links form a series inequality in S_1 with 39284 and a corresponding parallel inequality with S_2. The $-e$ link with S_2 is probably spurious, since 38497 will later be shown to be $D_{15}(2)$ (p. 371), and the link as shown is excessive. It is omitted in the diagram. No links are seen with S_3.

S (2).

(1) 21332				**1884·03**	(1) 29130		(5) 31726
1994·17							**1886·73**
(2) 23276	**3183·57** S_1	(4) 24062	**3183·70** S_2	**3183·38**	(1) 30430	**3183·11**	(3) 33613
		1940·68					**1940·93**
(5) 24578	**1881·95**	(2*n*) 26003					(2) 35554

Here is present a chain of e links in S_2 so frequent in Ag and Au [IV.]. Also, as in the case of $m = 1$, are found the same modification effect of about 2 in the u, v links. It may be noted that in addition to the above there are lines ($1n$) 26014·23 and ($3n$) 25295·58 respectively 1232·39 below S_2 and 1232·66 above 24062·92. Now $1231 \cdot 14 + \cdot 30\, d\Delta_2$ is the calculated value of a link e' constituted in the same way as e, but with Δ_2 in place of Δ_1.

S (3).

		(2) 39063	**1879·80**				
($2n$) 33791	$2\times$ **3182·59** S_1			$[S_2]$	**3180**	(1) 44123	
		(3) 39000	**1942·56**				

The line 40926 entered under S_2 is 769·80 ahead of S_1. The a link is 768·98, so that it may be $S_1(3)+a$. It is also numerically exactly the displaced line $S_2(3)(-9\delta)$, also $(\delta)S_2(3)$, but see below. Note that there is a rather abnormal $+e$ link with the normal value of S_2. The calculated value of S_3 is 41252·26 and is close on the limit of the region observed by Baly. Again with this, e' links are in evidence, viz., (2) 42483·95 and (1) 40023·21 respectively 1231·69 ahead and 1229·05 behind $[S_3]$, whilst the former has $\pm v$ links from it, viz., 1942·92 and $-1942 \cdot 97$. They are however not entered on the diagram, since only e, u, v links are there shown.

In the foregoing three orders the links are found in evidence. In the succeeding orders they are used as sounders.

S (4).

(1) 41919	**3178·11** S_1	(1) 42686	**3185·84** $(3\delta_1)S_2$	(3) 43000	**3183·04** $(2\delta_1)S_3$
1943·10					
(1) 39976		(1) 43987	**1884·63**		

The links here are unsatisfactory as well as the lines given for $S_{2,3}$. Numerically the second and third lines are respectively $(3\delta_1)S_2$ and $(2\delta_1)S_3$. They may really be displacements on the sequence terms, but if so the order is too high to make any certain decision. Taking S_1 as correct, the normal value for S_2 and S_3 would be 45884·31 and 46193·51. The former has links **1881·66** back to (1) 44002·65 with a further v, **1942·72**, back to (3) 42059·93. No direct link is found for normal S_3. Its $-e$ link should produce a line at 43010·16. This is very close to the mean of the 43000 shown in the table and a line (4) 43019·17, *i.e.*, 43009·87. In other words, these two lines are $(e)(2\delta_1)S_3$ and $(e)(-2\delta_1)S_3$, indicating that the normal S_3 line has been split into two by displacements $\pm 2\delta_1$ on the limit (or $\mp$ equal displacements in sequent). Both linked lines are seen, but only one of the lines themselves, which is possibly due to the fact that they are close on the limits of the observed region. We shall find indisputable evidence of such displaced parallel series as a general phenomenon.

S (5).

(1) 42293		
1884·65		
(2) 44237	**3181·73**	$[S_1]$
1944·48		
(3) 42352		

The separation $-e$ is given on the value calculated from the formula. In $m = 4$ the observed is 2·6 larger than the calculated, pointing to too small a value for the limit, which is quite possible as the limit determined from lines involving $S(1) = -P(1)$ is never found correctly. An increase of this order makes the $-e$ link in $S_1(5)$ normal. It gives the line in the table with $d\lambda = -\cdot 08$.

It is most important to get evidence which can carry conviction as to the reality of displaced lines, especially where the displacements occur simultaneously on both limit and sequent. A numerical coincidence in this case can have little weight by itself. In fact, it can have weight only in each particular case provided it is known from other considerations that such displacements are a common and universal rule. For this purpose it is instructive to adduce here some striking evidence afforded by certain sets of lines connected with the S series. The lines in question are arranged in the two following schemes, expressed in wave-numbers :—

−(1) 42844			−(3) 42059					−(8) 41496
123·00			**124·53**					**122·92**
−(2) 42721	**252·37**	$S_1(1)$	−(1) 41935	**252·94**	$S_2(1)$	−(1) 41625	**252·30**	$S'_3(1)$
300·11								
−(1) 42421								

(1*n*) 26407	**35·23**	(1) 26442	(2) 27175					
17·31		**17·29**	**22·53**					
(10) 26424	**35·31**	$S_1(2)$	(1) 27207	**38·70**	$S_2(2)$	(1) 27520	**35·17**	$[S'_3(2)]$
299·56		**299·72**	**298·15**					
(2) 26724	**35·47**	(5) 26759	(2*n*) 27506					

These lines are numerically displaced with reference to the S by amounts represented by the following parallel schemes :—

$S_1(1)(-9\delta)$		$S_2(1)(-9\delta)$			$S'_3(1)(-9\delta)$
$S_1(1)(-6\delta)$,	$S_1(1)$,	$S_2(1)(-6\delta)$,	$S_2(1)$,	$S'_3(1)(-6\delta)$,	$S'_3(1)$
$(-17\tfrac{1}{4}\delta)S_1(1)(-6\delta)$					

$S_1(2)(-9\delta)$,	$S_1(2)(-3\delta)$,	$S_2(2)(-9\delta)$			
$S_1(2)(-6\delta)$,	$S_1(2)$,	$S_2(2)(-6\delta)$,	$S_2(2)$,	$S'_3(2)(-6\delta)$,	$[S'_3(2)]$
$(-17\tfrac{1}{4}\delta)S_1(2)(-6\delta)$,	$(-17\tfrac{1}{4}\delta)S_1(2)$,	$(-17\tfrac{1}{4}\delta)S_2(2)(-6\delta)$.			

In addition, for $m = 3$, we have seen that in place of normal $S_2(3)$ the displaced $S_2(3)(-9\delta)$ is seen. The parallelism in spite of lacunæ show that the set are definitely related, and the fact that the same displacement on the sequents for $m = 1, 2, 3$ are required to represent the observed separations is specially striking, it being remembered that a displacement on the limit gives constant separation for different orders, whilst one on a sequent gives different for different orders. Here, for instance, 252 in $m = 1$, 35 in $m = 2$, and 16 in $m = 3$, all depend on the same oun multiple, 9δ, displacement in the sequences. Also 123 in $m = 1$ and 17 in $m = 2$ on the same 6δ, whilst the constant displacement 300 is explained by $-17\tfrac{1}{4}\delta = -69\delta_1$ on the limit.

Xenon.—Xenon also shows two spectra, without capacity in the red region and with it, extending far into the ultra-violet. Practically the only material at disposal is contained in the extensive lists of BALY* (red spectrum 6198—2536, blue 6097—2414) with an accuracy of about ·03 A. This is supplemented by observations by LIVEING and DEWAR,† especially by longer wave-lengths up to 6596, but unfortunately only measured to the nearest unit.

BALY draws attention to the large number of lines apparently common to both Kr and X. The number of lines in the whole spectrum is very large. BALY gives 1376 in the blue spectrum, but perhaps the most noticeable point for our present purpose is the great variability with change in the conditions of excitation. This is very clearly indicated by a comparison of intensities of corresponding lines as observed by L., D. and B. The following are a few examples out of a large number illustrating this. The numbers following a wave-length give the intensities as estimated respectively by BALY and by LIVEING and DEWAR :—

5191	5, 6	4890	5, 3
5188	4, 3	4887	5, 0
5080	7, 2	4884	1, 4
5068	not seen, 5	4883	6, 0
5045	3, 6	4844	10, 10

As between the two spectra also, a fact noticed by L. and D. is of importance. They say "there is one very remarkable change in the xenon spectrum produced by the introduction of a jar into the circuit. Without the jar the xenon gives two bright green rays at about λ4917 and λ4924, but on putting a jar into the circuit they are replaced by a single, still stronger, line at about 4922. In no other case have we noticed a change so striking." They also state that changes occur with the same kind of discharge as between different tubes. These are clear cases of oun displacements. PAULSON again (*loc. cit.*) gives some constant separations in the first spectrum. The triplet separations observed are about $\nu_1 = 1778$, $\nu_2 = 814$, in due order of magnitude with those for Kr. No line suitable for S (1) comes within the observed region, but there are two lines with W.N. 40375·40, 39561·50 separated by 813·99, which would serve for $S_2(1)$ and $S_3(1)$ and are in a similar position to the KrS lines. They clearly suggest that the $S_1(1)$ line is at $-42153·39$, using $\nu_1 = 1777·90$ as found from the S_2 set. This is further substantiated by employing the value of the *e* link, found below to be 7314, as a sounder. It requires a line at about 34839, and such a line is found at (<1) 34836·78 (but see discussion under

* 'Phil. Trans.,' A, vol. 202, p. 183 (1903).

† 'Roy. Soc. Proc.,' vol. 68, p. 389 (1901); 'Coll. Papers,' p. 494.

$S_3(3)$). The observed S sets, as well as others found by sounding, are given in the following table:—

XS.

m.	S_1.		S_2.		S_3.
1.	–[2371·57] [42153·39]	**1777·90**	–(10) 2476·02 40375·49	**813·99**	–(4) 2526·97 39561·50
2.	(4) 3854·44 25936·90	**1777·90**	(5) 3607·17 27714·80	**815·11**	(1) 3503·99 28530·91
3.	(4) 2527·10 39559·46	**1776·69**	(1) 2418·47 41336·15	**814·63**	(<1) 2869·71.*e* (42150·88)
4.	(1*n*) 2689·82.*e* (44480·53)	**1777·44**	(4) 2871·85.*e.u* (46257·97)	**817·68**	(1) 2829·35.*e.v* (47075·65)
5.	(1) 2828·01.*e.u* (46797·58)	**1779·33**	(1) 2944·78.2*e* (48576·91)		
6.	(2) 2452·76.*e* (48072·37)	**1778·33**	(1*n*) 2623·31.*e.v* (49850·70)	**815·34**	(1) 2549·05.*e.u* (50666·04)
7.	(<1) 2921·74.*e.e* $(-\delta_1)$ (48866·68)	**1777·80**	(4) 2777·10.*e.e* (δ_1) (50624·48)	**815·22**	(1) 2715·91.*e.e* $(-\delta_1)$ (51439·70)
8.	(2) 2658·37.*e* $(-\delta_1)$.*v* (49350·39)	**1778·69**	(1) 2538·16.*e.v* (51129·08)	**816·00**	(2*n*) 2468·54.*e.u* (51945·08)
9.	(1) 2850·41.2*e* (49700·78)				
10.	(1) 2616·79.*e* $(-\delta_1)$.*v* $(-\delta_1)$ (49949·59)	**1777·55**	(1) 2701·99.2*e* (51727·14)	**815·61**	(1) 2432·87.*e* $(-\delta_1)$.*u* $(-\delta_1)$ (52542·75)
11.	(1) 2584·04.*e.u* (50134·99)	**1781·25**	(2) 2470·30.*e.u* (51916·24)	**813·57**	(1) 2943·07.2*e.u* (52729·81)
12.	(2) 3202·17.2*e.v* (50276·15)	**1777·00**	(1) 2479·98.*e.v* (52053·14)	**817·59**	(3) 2614·13.2*e* (52870·73)
13.			(2) 2663·43.2*e* (52162·72)	**815·00**	(<1) 2921·74.2*e.u* (52977·72)

The first three lines gives the formula

$$n = 51025{\cdot}29 - \mathrm{N} \Big/ \left\{m + {\cdot}096726 - \frac{011826}{m}\right\}^2.$$

From the limit 51025, and using the observed separations given by the S (2) lines, viz., 1777·90, 815·05, the values of Δ_1, Δ_2 are found to be

$$\Delta_1 = 24893 + 13{\cdot}64\, d\nu_1 - {\cdot}72\xi = 40\tfrac{3}{4}\{610{\cdot}87 + {\cdot}334\, d\nu_1 - {\cdot}018\xi\},$$

$$\Delta_2 = 10996 + 13{\cdot}33\, d\nu_2 - {\cdot}31\xi = 18\ \{610{\cdot}89 + {\cdot}74\, d\nu_2 - {\cdot}017\xi\}.$$

To a first approximation therefore the oun is given by $\delta = 610{\cdot}88$. The calculated value of the *e* link from the value of Δ_1 is $e = 7314{\cdot}09 - {\cdot}0056\xi + 4{\cdot}23\, d\nu_1$. The result of examining the lines of the blue spectrum for separations of this magnitude is shown in Plate 2, fig. 2. It is distinguished from those of elements hitherto discussed by showing one definite maximum alone at about 7315·3, although there are indications of the appearance of another peak beyond 7317. The displaced $e(\delta_1)$ link shows a difference of 2·32, so that a second peak might be expected at 7317·62. If the actual value is at 7315·3 it would require $4·23\,d\nu_1 = 1·2$, or $d\nu_1$ about ·30, $d\lambda = ·04$ distributed between the two $S_{1,2}(2)$ lines. This is possible with ±·02 on each line, but probably excessive for an error on one of them. Both values are tested as links below for the observed lines, and the results show that with the exception of $S_1(1)$ the value of e, calculated from the original ν_1, is extraordinarily exact. For this reason, and because the exact position of the peak of the frequency curve depends on several disturbing conditions, the original value $e = 7314·1$ will be used for sounding purposes on lines outside observed regions. Again the first line −42153 and the limit 51025 gives 93178 as the value of $P(\infty)$ or $s(1)$. From this the values of the u, v links are found. The complete set are

Links.	Changes per δ_1 displacement.
$u = 4133·18 - ·049\xi + 2·19\,d\nu_1$	1·77,
$v = 4428·00 - ·061\xi + 2·51\,d\nu_1$	1·84,
$e = 7314·1 - ·0056 + 4·23\,d\nu_1$	2·32.

The results obtained by sounding are shown at a glance in diagram Plate 3, which embraces orders up to $m = 13$. The cumulative weight of the evidence is overwhelming in support of the general application of this method. The existence of a series parallel to the normal S at a distance $-e$ is proved, whilst the presence of other linked lines is rendered extremely probable by succession of similar linking in the same set, and in neighbouring orders. Compare, for instance, the triplets in $m = 1, 3, 4, 7, 8$ and the sets for $m = 8, 9, 10$.

Detailed Discussion.—In the following discussion the starting point for the consideration of each triplet set is—after $m = 3$—the value of $S_1(m)$ calculated from the series formula obtained above. The sounders are indicated to the left of each observed line and the values O−C in $d\lambda$ are given on the right, the observed or O line being regarded as the observed sounded line + the link as given above. The value entered in the table of S lines above is, however, not the line as calculated from the formula, but the most probable value as deduced from sounding. They are indicated below by asterisks. For the first three orders the values of $d\lambda$ obtained by using $e = 7315·3$ are placed to the right of those depending on $e = 7314·1$.

S (1).

[−42153·39]			−40375·49			−39561·50		
$-e(<1)$ 34836	·15,	·07	$-e(5)$ 33061	·01,	−·06	$-e(3)$ 32248	−·07,	−·14
$-2e(4)$ 27523	·13,	−·02	$-2e(1)$ 25747	·00,	−·15	$-e-u(1)$ 28113	·04,	−·07
$-e-u(3)$ 30700	·37,	·11	$-e-u(1)$ 28928	·00,	−·11	$-e+u(3)$ 36379	·11,	·05
						$-e-v(1)$ 27819	·00,	−·12

In the case of S_1 it appears as if the $e = 7315$ is much superior. But, as it happens, the value of $S_3(3)$ as calculated from $S_2(3)$ by ν_2 is 42151·20, and so is very close to $S_1(1)$. It will be shown that for this the 7314 link gives very close values, and the linkage probably belongs to $S_3(3)$. The $-e-u$ is doubtful.

S (2).

25936		27714		28530	
$-e(3)$ 18622	·00, −·19	$-v(2)$ 23282	·51	$v(2)$ 32958	·00, ·17
$-e+u(10)$ 22755	·08, ·16	$-v(-2\delta_1)$	·00		
$-2e$ [15441·84]					

In S_1 the intensity of 22755 would suggest that its linkage is a coincidence. L.D. give a line 15447 which may possibly be $(-2e)S_1$, for their measures are only to the nearest A.U.

S (3).

39559	41336		[42151]	
See $S_3(1)$	$-e(<1)$ 34022	−·03, −·10	$-e(<1)$ 34836†	·02, −·05
	$-e+u(<1)$ 38155	−·03, −·06	$-2e(4)$ 27523	·00, −·17
	$-e-u(1)$ 29892	−·20, −·31	$-e-u(3)$ 30700	·22, ·14
	$-e+v(<1)$ 38453	−·16, −·22	$-e-u(-2\delta_1)$,,	·01
	$-2e+u(1)$ 30843	−·14, −·16		

† $S_1(1)$ and $S_3(3)$ are very close. This sounder probably is correct here and does not hold for $S_1(1)$, for which it differs by about 3.

S (4).

[44481·06]		[46258·43]		[47073·48]	
$-e(1n)$ 37166	·02 *	$-e(1)$ 38940	·16	$-e(1n)$ 39763	−·15
				or (<1) 39754	·23
$-e-u(3)$ 33030	·17	$-2e(1)$ 31628	·09	$-e-v(1)$ 35333	−·08 *
$-e-u(-2\delta_1)$,,	·00	$-e-u(4)$ 34810	·02 *		
		$-e-v(<1)$ 34518	−·08		

The 38940, 39763 are too far out to be dependent on e links. They could be dependent respectively on $e(-2\delta_1)$ and $e(+2\delta_1)$. Or more probably the lines 38940, 39754 may be $S_2(4)(-2\delta).e$, $S_3(4)(-2\delta).e$.

S (5).

[46799·33]		[48577·23]		[49392·28]
$-e(1n)$ 39491	·28	$-e(1)$ 41257	−·23	
$-e-u(1)$ 35350	·08 *	$-e-v(<1)$ 36832	·11	
		$-2e(1)$ 33948	·01 *	

39491 is too far out for a link. It would give a reading for $S_1 = 46805·45$. It is curious, however, to notice that we have sounders for a set with this value, viz. :—

46805·45	**1779·80**	48585·01	**815·52**	49400·53
$-e(1n)$ 39491		$-e(3)$ 41271·15		$-e-u(1)$ 37593·25

S (6).

[48072·77]		[49850·67]		[56665·97]	
$-e(2)$ 40758	·02 *	$-e$ [42536]		$-e$ [43352]	
		$-e-v(1n)$ 38108	·00 *	$-e-u(1)$ 39218	·00 *

S (7).

[48846·60]			[50624·50]			[51439·80]		
$-e$[41532]			$-e$[43310]			$-e$[44125]		
$-2e(<1)$ 34216	·08	*	$-2e(4)$ 35998	$-$·08	*	$-2e(1)$ 36809	·08	*
$-e-v(2)$ 37105	$-$·02							

There seems clear evidence of displacement here producing a separation of about 2, which is the same as that by δ_1 on e. The $-2e$ soundings give $S_3-S_1 = 1777 \cdot 90+815 \cdot 12$, which is correct, but 35998 is 2 in the opposite direction. The lines in the list are therefore deduced from this set, using as sounders $e+e(-\delta_1)$, $e+e(\delta_1)$, $e+e(-\delta_1)$. The values of $d\lambda = \cdot 00$ for the set.

S (8).

[49351·72]			[51129·62]			[51944·92]		
$-e$[42037]			$-e$[43815]			$-e$[44630]		
$-e-v(2)$ 37606	·14		$-e-v(1)$ 39386	·02	*	$-e-v(1)$ 40199	·12	
$-e-u(3)$ 37901	·11		$-e-u(3n)$ 39683	$-$·06		$-e-u(2n)$ 40497	00	*
$-2v(2n)$ 40497	$-$·11							

The linkage separation gives for the $-e-v$ **1780·83+812·41** $= \nu_1+\nu_2+\cdot 29$.

" " " " $-e-u$ **1782·19+813·95** $= \nu_1+\nu_2+3 \cdot 19$.

The separations about 1780 are very common and will be discussed more completely under the D series, but here the origin must be a different one. For S (1) the sounder $-e(-\delta_1)-u$ is taken.

The $-2v$ link for S_1 is probably spurious. The line comes under S_3 as well. This is because of the numerical coincidence $\nu_1+\nu_2+2v = 11448 \cdot 95$ and $e+u = 11447 \cdot 28$. It appears in $m = 9$, 10 also.

S (9).

[49699·55]			[51477·45]			[52292·75]		
$-e$[42385]			$-e$[44163]			$-e$[44978]		
$-2e(1)$ 35072	$-$·05	*	$-2e(1)$ 36842	·24		$-e-u(1)$ 40841	·16	
$-e-v(1)$ 37953	·18							
$-2v(1)$ 40841	·10							

Only one reliable value $-2e$ for S_1.

S (10).

[49949·21]			[51727·11]			[52542·41]		
$-e$[42635]			$-e$[44313]			$-e$[45228]		
$-e-v(1)$ 38203	·16	*	$-2e(1)$ 36998	·00	*	$-2e(2)$ 37916	·17	
$-2v(1)$ 41091	·07					$-e-u(1)$ 41091·56	·13	*

The $-e-v$ in S_1, $-2e$ in S_2, eu in S_3 give $1781 \cdot 53+811 \cdot 70 = \nu_1+\nu_2+\cdot 28$ the modified ν_1. For S_1 and S_3 the modified $e(-\delta_1)$, $u(-\delta_1)$, $v(-\delta_1)$ are taken. The line 37916 under S_3 is also $(ev) D_{16}(8)$, considered later.

S (11).

[50134·46]			[51912·36]			[52727·66]		
$-e$[42820]			$-e$[44598]			$-e$[45413]		
$-e-u(1)$ 38687	$-$·02	*	$-e-u(2)$ 40468	$-$·28		$-2e-u(1)$ 33968	$-$·08	
$-2e-v(<1n)$ 31076	·05					$-2e-v(3)$ 33674	$-$·12	

Again the modified $\nu_1 = 1780$ with $eu\,(S_1)\,euS_2$. These and $(2e,\,u)\,S_3$ give

$$1781\cdot25+813\cdot57 = \nu_1+\nu_2-1\cdot37.$$

S (12).

[50275·68]			[52053·58]			[52868·88]		
$-e$ [42961]			$-e$ [44739]			$-e$ [45554]		
$-2e-v$ (2) 31219	$-\cdot02$	*	$-e-v$ (1) 40311	·01	*	$-2e$ (3) 38242	$-\cdot07$	*

S (13).

[50385·80]	[52163·70]		[52979·00]	
	$-2e$ (2) 37534	·03	$-2e-u$ (<1) 34216	·04

These higher orders are necessarily close to high orders of the D series, and many are therefore apparently common to both. *E.g.*, 34216 has been adduced as $(2e)\,S_1\,(7)$ and is also connected with a D line. Also their wave-numbers are now so high that it requires two sounders in series to just reach the limits of the observed region. The later identifications are therefore all doubtful.

Argon.—For the red or non-condensed spark spectrum about 360 lines between 8015 and 2476 together with 16 lines in the ultra-red have been observed. For the blue or condensed spark spectrum the number amounts to about 780 between 6682 and 2050 together with another 40 lines in the extreme ultra-violet between 1886 and 1333. The measures in the red are due to PASCHEN* (ultra-red), KAYSER,† RUNGE and PASCHEN,‡ and EDER and VALENTA,§ and in the blue to KAYSER,† EDER and VALENTA,§ and LYMAN|| (extreme ultra-violet). In addition we have exact interferential measures in I.A. by MEISSNER¶ for some red lines and measures by BALY** for a few extra lines in the blue spectrum. The red spectrum is noted for the existence of the sets of constant separations discovered by RYDBERG.†† The present communication, however, deals chiefly with the blue spectrum.

The search for the S series in A is more difficult than in the cases of Kr and X. There are an extremely large number of separations of about the same value but clearly distinct. They range round 179 to 188, and, as will be seen later, displacements are very common. The clue is given from the analogous S (1) lines for Kr and X. The only strong triplet lines in the corresponding positions are those given in the following list :—

* 'Ann. d. Phys.,' vol. 27, p. 537 (1908).

† 'Berl. Ber.' (1896), p. 551; 'Astro. Journ.,' vol. 4, p. 1 (1896).

‡ 'Astro. Journ.,' vol. 8.

§ 'Denks. Wien. Akad.,' vol. 64, p. 216 (1896).

|| 'Astro. Journ.,' vol. 33, p. 107 (1911).

¶ 'Ann. d. Phys.,' vol. 51, p. 95 (1916).

** 'Phil. Trans.,' A, vol. 202, p. 188 (1904).

†† 'Astro. Journ.,' vol. 6, p. 338 (1897).

AS.

m.					
1.	−(5) 2344·4 42642·10	**179·43**	−(3) 2354·3 42462·67	**75·60**	−(1) 2358·5 42387·07
2.	(5) 3765·463 26549·76	**181·64**	(2) 3739·88 26731·40	**74·74**	(9) 3729·450 26806·14
3.	(1) 2484·1 40244·05	**179·10**	(2) 2473·1 40423·15	**70·39** **75·59**	(1) 2468·8 40493·54 (40498·74)
4.	(2212·7) (45179·30)	**179·50**	(2204·0) (45358·80)	**75·60**	(2200·3) (45434·40)
5.			(2098·5) (47681)		
6.	[2049·5] [48776·74]		[2042·2] [48950·34]		[2038·8] [49031·94]

The observation errors for $m = 1$ and 3 are very considerable, since the measures are only given to ·1 A.U. and ·05 A. produces about $dn = $ ·8. Consequently it is possible only to obtain approximate values for ν_1, ν_2. On the other hand, for $m = 2$, where we have very accurate measures, there must be some doubt about the allocation of $S_3(2)$ because its intensity, 9, is so excessive in comparison with the 5, 2 of S_1 and S_2, and the ν_1 separation of 181·64 is greater than observation errors on the lines for $m = 1$, 3 allow. The latter objection, however, can be set aside as it corresponds to the excess ν_1 observed in Kr and X diffuse sets and, as will be found later, in NeS. In these cases $\nu_1+\nu_2$ comes out to be normal. Here, however, the sum is about 1·35 ± too large, and with the S(1) separations the typical $S_3(2)$ would be at 26804·79 or $d\lambda = $ ·18, probably of intensity 1, and so overshadowed by the strong line in the list. As will be seen immediately, the linkages will show that this value is preferable. The linkages will also show the probability of a line at 40498 for $S_3(3)$.

The three first S_1 lines give the formula

$$n = 51731{\cdot}05 - \mathrm{N}\Big/\left\{m + {\cdot}095901 - \frac{{\cdot}017878}{m}\right\}^2.$$

For the determination of Δ_1, Δ_2, the oun and the links, reliable values of ν_1 and ν_2 are required. We have seen that the values obtained from the observed sets of lines are subject to large observational errors. Nevertheless that the true value of ν_1 is not far from that shown by S(1) is indicated by the fact that there are several accurate separations of about 179·5, *e.g.*,

(6) 20621	**179·31**	(8) 20800	**179·61**	(4) 20980
(3) 26893	**179·35**	(1) 27072		
(1) 31359	**179·62**	(4) 31538		

of which the first is part of a linkage. With the limit $51731{\cdot}05+\xi$ and $\nu_1 = 179{\cdot}50+d\nu_1$, $\nu_2 = 75{\cdot}60+d\nu_2$, the values of Δ_1, Δ_2 are

$$\Delta_1 = 2519-{\cdot}073\xi+14\,d\nu_1 = 43\tfrac{3}{4}(57{\cdot}59+{\cdot}32\,d\nu_1-{\cdot}0016\xi),$$

$$\Delta_2 = 1057-{\cdot}030\xi+14\,d\nu_2 = 18\tfrac{1}{4}(57{\cdot}91+{\cdot}77\,d\nu_2-{\cdot}0016\xi),$$

or

$$\Delta = \Delta_1+\Delta_2 = 62\times 57{\cdot}70.$$

The oun is thus given by $4\delta_1 = \delta = 57{\cdot}7$ with some uncertainty owing to inexactness in the observed triplet separations. Its value calculated direct from the atomic weight 39·9 should be $\delta = 361{\cdot}8\times({\cdot}399)^2 = 57{\cdot}6\pm{\cdot}14$, the uncertainty being due to the uncertainty $\pm{\cdot}05$ in the atomic weight. The value of $s(1) = p(1)-\mathrm{S}(1) = 94373{\cdot}10$, from which the u, v links may be calculated. The e, u, v links are found to be

$$e = 719{\cdot}71+4\,d\nu_1,$$
$$u = 439{\cdot}47+2{\cdot}43\,d\nu_1,$$
$$v = 442{\cdot}67+2{\cdot}47\,d\nu_1.$$

The examination of the spectrum gives the occurrency curve shown in Plate 2, fig. 4. It shows a very distinct maximum in the region around 720 but little to show the exact position. The values appear somewhat irregular. If, *e.g.*, the ordinate for 719·6 be drawn, it would be the same as for those at 720·5 and 721. It may be noted that if the maximum is taken at 720·4, ν_1 is ·18 larger = 179·68 and the oun calculated from Δ_1 becomes the same as from Δ. But this is rarely the case in triplets. As, moreover, the link 719·7 when applied to the observed S(1, 2, 3) lines gives better agreement than a link 720·4 we shall use it for the purpose of sounding.

The results of sounding are exhibited in diagram form in Plate 3. The details follow :—

S(1).

42642·10		42462·67		42387·07	
$-e$[41922·39]		$-e$[41742·96]		$-e$(3) 41666	−·06
$(-7\delta_1)$(2) 41474		$-e-v$(4) 41299	·02	$(-7\delta_1)$(2) 41218	
$-e-u$ 81·86	·06			$-e-v$	−·03
$(6\delta_1)$(1) 41488		$-2e$(2) 41023	−·02	$(6\delta_1)$(2) 41231	
$e(-6\delta_1)$(1) 43355	·00	$-2e+u$(4) 41460	·11	$-2e$(1) 40949	−·10
		$-2e-u$(2) 40585	−·10		
		$+e$(3) 43183	−·06		

We find in the three sets clear indications of displacements producing separations of 12 or 13. As they appear in different orders such displacements can only arise in the limit. Two cases occur in $m = 1$, viz.,

$S_1(1)-e-u$ and $S_3(1)-e-v$. They land between the lines indicated above, where the separations are 13·76 and 13·59, or the same within error limits. Now $13\delta_1$ on $S(\infty)$ produces 13·26, and the four lines in question are $41474{\cdot}51 = (7\delta_1)S_1(1)-e-u$ or $e.u.(7\delta_1)S_1(1)$, $41488{\cdot}27 = e.u.(-6\delta_1)S_1(1)$, $41218{\cdot}08 = e.v.(7\delta_1)S_3(1)$, $41231{\cdot}67 = e.v.(6\delta_1)S_3(1)$. That is, S_3 repeated from S_1.

S (2).

26549·76		26731·40		26806·14	
$(-3\delta)(4)$ 25817		$(-6\delta_1)(1)$ 27165		$u-5{\cdot}8(1)$ 27239·82	
$-e$ 29·44	·09	u 71·50	−·09		
$(2\delta)(7)$ 25841		$(6\delta_1)(2)$ 27177		$v-6{\cdot}2(1)$ 27242·58	
$2e-v(4)$ 24665	·34	$-u(-7\delta_1)(3)$ 26285	−·05	$-e-12(3)$ 26098·42	
$-e-u(1)$ 25391	−·17	$-(3)$ 26001		$-2e+u(1)$ 25804	·31
		$-e$ 11·98	−·04		
		(3) 22		$-2e+u+v(3)$ 26248	·11
		$-2e(1)$ 25290	·14	$-2e-v(1)$ 24921	
		$-2e-v(3)$ 24845	·58		

Several interesting points emerge from the above.

(1) The links to 26806 as S_3 are all bad. On the other hand, if the links are regarded as correct, the last three point back respectively to 26803·97, 05·34, 03·35, or a mean 26804·22. If S_1 is correct and $\nu_1+\nu_2 = 179{\cdot}50+75{\cdot}60$ as assumed S_3 should be 26804·86, the same within the various error limits. This is therefore supporting evidence in favour of the supposition made above that the very strong line (9) 26806 is not itself $S_3(2)$, but that it overshadows the true one, which ought to be a very weak one.

(2) The existence of separations in the neighbourhood of 12 is also very marked. There are lines (4) 26562·08 at 12·22 above S_1, (2) 26744·15 at 12·75 above S_2, and (3) 26098·42 at 12 above the linked line S_3+e. Further, 12, 2×12, appear in connection with the different linked systems, showing the existence of sets of lines depending on displacements in $S(\infty)$.

(3) In S_2 the allotted line is 181·64 ahead of S_1 instead of the normal 179·50. The majority of the links from S_1 are to displaced lines, in which δ_1 gives a separation of about 1·03, so that no direct evidence as to normal value of S_2 is directly available, beyond the fact itself that displacement is very prevalent. The two only direct links, $2e$, $2e+v$, would refer back to lines at 26730·34, 27·77, or a mean of 26729·05, which is 179·3 ahead of S_1 and clearly points to the normal S_2. In other words, the normal S_2 is displaced by $(-2\delta_1)S(\infty)$, but the linked lines refer back to the undisplaced normal lines. With the development of our knowledge of the laws of spectral construction, such facts as these may be expected to be of the greatest importance in settling questions of internal constitution of individual vibrating systems.

S (3).

40244		40423		(40498·74)	
$e+6(3)$ 40969		$e+7{\cdot}3(1)$ 41150		$e(2)$ 41218	·02 *
$e+v(3)$ 41407	−·04	$2e+6{\cdot}4-v(3)$ 41426		$e+v(2)$ 41860	·04
$2e(3)$ 41681	·07	$2e(2)$ 41860	·14	$2e(3)$ 41940	−·16
$2e-v(3)$ 41243	·01	$-e-u(1)$ 39265	−·12	(1) 41488	
				$2e-u$ 500	−·10
$-e+12(4)$ 39536	·02	$-v(5)$ 39981	−·08	(1) 41512	
$-2e+u(1)$ 39244	−·01	$-2e+u(3)$ 39420	·16	(2) 40926	
				u 37	·02
				(1) 40949	

Here again the links to the line observed near normal S_3 are bad, whereas the links treated as good refer back to a line exactly ν_2 ahead of S_2. Further the 12 or 6 separation is again in evidence. Also the links $2e$, $-2e+u$, $-e-u$ refer back to lines differing by about 6 from S_2.

S (4).

[45179·30]		[45358]		[45434·40]	
u (1) 45618	−·02	$e-v$ (1) 45635	·00	$-e-u$ (1) 44996	−·09
$-v$ (4) 44735	·07	$-v$ (3) 44913	·12	$-e-u$ (1) 44275	−·01
$-2e+v$ (2) 44175	−·16				

Here the calculated lines agree remarkably with the sounded. The calculated are therefore adopted.

S (5).

[47501·68]	[47681·18]		[47756·78]
	v (1) 48126	−·12	
	$u+6$ (1) 48126	·00	

The only links apparent are for S_2, again with the 6 displacement. These lines are all close to the limit of observed region.

S (6).

[48776·74]		[48956·34]		[49031·94]	
$-2e+v$ (1) 47784	·16	$-2e-12$ (1) 47522	·09	$-e+u$ (1) 48753	·06
				$-2e-u$ (1) 47155	−·10

The lines are now so far out of the observed region that the links are too small to refer back to well observed regions even if such lines are really existent.

The foregoing discussion of the S series affords evidence of the existence of displacements of about 12 (or 6). This requires further consideration as affording material on which additional knowledge may be obtained regarding the laws which such displacements follow. The presence of the same displacements in successive terms of the series points to a modification on the limit—either a pure displacement, a linkage effect, or, as the separation is small, possibly the difference of two links. The further evidence to follow points decisively to the existence of displacement. Whether they are due to displacements by multiples of the oun on the limit is, of course, not so convincing. The numerical relations are very closely represented on this hypothesis, but in the case of argon the δ_1 is so small that it produces in S_1 separations of 1·03 only. In lines whose wave-numbers lie about 40,000 or greater, this produces changes in λ of ·06 and therefore comparable with observation errors. In the case of S (2) only—wave-numbers of order 26700—does it produce $d\lambda = $ ·15. The measurements here are by KAYSER, whose errors are probably $<$ ·02, almost certainly $<$ ·05, and a close agreement between calculated and observed lines will give evidence of some weight.

$m = 1$. In S (1) cases of the displacement associated with the linked lines have been given above. They also exist in connection with the S lines themselves. Near $S_2(1)$ are 42448·25 and 42473·49, both of intensity 1; the first 14·42 above and the second 10·82 below $S_2(1)$, or a difference of $25{\cdot}24 = 2 \times 12{\cdot}62$. Again (3) 42401·45 is 14·38 above $S_3(1)$. Now $3\frac{1}{2}\delta = 14\delta_1$ displacement on S (∞) produces a separation of 14·42, and of $2\frac{1}{2}\delta = 10\delta_1$ one of 10·30. If, therefore, the observation errors are very small the lines in question are respectively $(-14\delta_1)\ S_2(1)$, $(10\delta_1)\ S_2(1)$, and $(-14\delta_1)\ S_3(1)$. There is no corresponding $(-14\delta_1)\ S_1(1)$, but (1) 42623·93 is $(-18\delta_1)\ S_1(1)$ with $d\lambda = {\cdot}00$.

$m = 2$. In S (2) we find direct displacements on the S (2) lines with a whole set of linked lines. They are indicated in the following table :—

						(9) 26806·14	
(4) 26562·08		**182·07**	(3) 26744·15		**75·60**		19·75
						(1) 26833·37	
$-e$ (7) 25841	−·13		$-e$ (3) 26022			$-e$ (3) 26098	
$-2e$ (5) 25121	−·09		e (2) 27464	−·12		$-e+24{\cdot}61$ (3) 26123	
$-3e$ (1) 24399	·30		$-u$ (2) 26305	−·12			
$-u$ (3) 26123	·06		$-e-u$ (1) 25582				
$-e+v$ (3) 26285	·00						
$-3e+v$ (3) 24845	·00						

The line 26562 has been already adduced. It is $(-2\delta)\ S_1(2)$ with $d\lambda = -{\cdot}02$. The lines 26022, 26098 under $(e)\,S_2$ and $(e)\,S_3$ differ by 75·57 or a normal ν_2, and refer back to lines which would give the true triplet separations with the first line 26542.

$m = 3$. From $S_1(3)$ the two lines (1) 40258·62 and (4) 40273·21 are displaced successively by 14·57 and 14·59, the same separation as in the case of $m = 1$. They are $(-14\delta_1)\ S_1(3)$ and $(-28\delta_1)\ S_1(3)$, with O−C given by $d\lambda = {\cdot}00$ in both cases. The first has a modified e link −722·61 and the second another of 718·46 to observed lines (4) 39536·01; (6) 40991·67.

The D *and* F *Series.*

It will be found that the rare gases show a large preponderance of sets of lines which have all the characteristics of belonging to satellites of D series, or to parallel multiplet F series. In other words where, as in other elements, the d sequences show two displacements from the main d_{11} sequence when triplets are in question, the rare gases show a large number of such displacements, both in the d and the f sequences, many of these being due to large multiples of Δ_2. No attempt has been made to determine the whole system of these satellite sets, a problem belonging to an intensive study of individual elements, but a sufficient number have been adduced to prove their existence and to exhibit some of their characteristics.

Krypton.—The following table contains lines allocated to the D series for krypton and discussed in the present communication:—

KrD.

$m = 1$	D_{18}	(1) 19116·44	**788·25**	(2) 19904·69	**309·71**	(2) 20213·86
			790·47	(1) 19906·91	**306·95**	
		1860·19				
	D_{17}	(1) 19928·46	**786·03**	(1) 20714·49	**311·84**	(2) 21026·33
			789·67	(3*n*) 20718·13	**308·20**	
		1048·17				
	D_{16}	(2) 20669·36	**788·40**	(5) 21457·76	**308·82**	(3*n*) 21766·58
		307·27				
	D_{15}	[20763·25]	**789·40**	(2) 21552·65	**306·33**	(2) 21858·98
		213·38				
	D_{14}	(2*n*) 20842·89	**788·78**	(1) 21631·67	**307·68**	(4) 21939·35
		133·74				
	D_{13}	(1) 20871·60	**788·75**	(5) 21660·35		
		105·03				
	D_{12}	(<1) 20875·78	**788·28**	(2) 21664·06		
		100·85				
	D_{11}	(6) 20976·63				
$m = 2$	D_{18}	(1) 38037·85	**787·60**	(1) 38825·45	**311·96**	(2) 39137·41 ?
	D_{17}	[38275·82]	**787·44**	(2) 39063·26		?
		[38324·67]	**787·33**	(2) 39112·00	**308·99**	(1) 39420·99
	D_{15}	(1*n*) 38497·23	**787·16**	(1) 39284·39	**306·87**	(2) 39591·26
	D_{13}	(1) 38531·94		?		
	D_{11}	(5) 38560·46				
$m = 3$	D_{15}	(2) 44408·53	**787·99**	(2) 45196·52	**308·07**	(45504·59)
	D_{11}	(3) 44426·67				
$m = 4$		(47077·38)	**786·19**	(47863·57)	**308·15**	(48171·72)
		(47080·55)				
$m = 5$		(48500·71)	**786·88**	(49287·59)	**305**	[49593·55]
$m = 6$		(49345·98)	**787·97**	(50133·95)	**309·89**	(50443·84)
		[49349·39]				
$m = 7$		(49896·60)				

It will be clearer to consider first the D_{11} lines by themselves, and then the satellite lines in each order. From the first three lines the calculated formula is

$$n = 51655{\cdot}56 - N \Big/ \left\{ m + {\cdot}897262 - \frac{{\cdot}006513}{m} \right\}^2.$$

The limit is within errors of the value found for $S(\infty)$, but in view of what happens in the cases of X, and RaEm to be considered later, it may be noted that the difference $D_\infty - S_\infty = 4{\cdot}27$ is very close to a $-\delta_1$ displacement in $S_1(\infty)$, which would produce a separation of 4·42. [*Note.*—If $D(\infty) = S(\infty)$, ξ here $= -4{\cdot}27 + \xi$

of S (∞). If D (∞) = $(-\delta_1)$ S (∞), ξ here = $\xi -$·15 of S (∞).] The lines calculated from the formula for the succeeding lines are all outside the observed region. For $m = 4 \ldots 7$ they are respectively in wave-numbers

47079·52, 48500·56, 49349·39, 49896·60,

The detailed discussion immediately following will show the evidence for their existence by sounding. The existence of a parallel set at a distance $-(e+v)$ is brought to light which show for the above calculated lines values of O−C = ·09, ·00, ·14, −·01.

The agreement is remarkably close and would seem to show that the value of D (∞) used is very close to its true value, about one or two units in excess. The results of the discussion are exhibited as a whole in Plate 3, fig. 4.

As in the previous cases, the linkings with the observed lines m = 1, 2, 3 are given, in order to show that the method is justified where it can be tested.

1.

(6) 20976	
e (4) 24158	·17
v (1) 22914	·86
$v\,(-\delta_1)$ 22914	·00
$-e-v$ (3) 15856	

2.

(5) 38560	
e (1) 41743	·04
$-v$ (1) 36619	·27

3.

(3) 44426	
$-v$ (10) 42483	·01
$-2e$ (4) 37061	·13

4.

[47079·52]	
$-e$ (7) 43894	·08
$-e-v$ (1) 41951	·09
$-u$ (2) 45196	·00

5.

[48500·56]	
$-e$ [45317]	
$-e-v$ (1) 43374	·00

6.

[49349·39]	
$-e$ [46166]	
$-e-v$ (2) 44220	·13

7.

[49896·60]	
$-e$ [46713]	
$-e-v$ (1) 44771	−·01
$-2e-v$ (3) 41588	·05

In m = 1 negative links lead to lines in the red where observation is defective. LIVEING and DEWAR give a line to the nearest unit which may possibly be a $-e-v$ link. The line 22914 is too far out to be an exact v, but its difference is 4·47, which corresponds to an exact $-\delta_1$ displacement on the limit. In other words, it belongs to the limit as calculated from the S series.

In $m = 4 \ldots 7$ the $-e-v$ sounders all show lines in evidence. The $-e$ sounder for m = 5, 6 would show lines just within the boundary of observation, and the absence of corresponding observed lines is therefore explicable. In m = 4 the line 41951 is part of an apparent triplet.

(1) 41951·59 **786·19** (1) 42737·78 **308·51** (2) 43045·93.

The diminished values of ν_1, ν_2 indicate that the second and third lines correspond to D_{22} and D_{33} lines. The value of ν_1 is however sufficiently close to make the difference due to observation error, in which case

41951 would correspond to a D_{12}, leaving the calculated 47079 as a true D_{11} ($d\lambda = 0$), whilst D_{12} is 47077·38. That 47079 exists is also shown by the u sounder. In the table this arrangement is adopted. The line entered for $m = 5$ is the calculated, as it is so close to the deduced. For $m = 6, 7$ they are the deduced from the $-e-v$ sounder. The sounders for D_2, D_3, $m = 4 \ldots 7$, are indicated in Plate 3, fig. 4.

The Abnormal Satellite Separations.—The separations of the lines suggested for the satellite sets show abnormal values in that they are roughly about 2 greater than ν_1 for the S triplets. The difference is real and not due to errors of observation, and we shall find a corresponding abnormality in the other elements of the group. Taking BALY's maximum error to be $d\lambda = \cdot 05$, the maximum error in n for the D (1) lines will range from ·21 to ·24, or, say, ·45 on a difference of two lines. All the D (1) readings for ν_1 can therefore be the same within observation errors, but cannot possibly agree with that for the S set. Those for ν_2 however, 308·82, 307·68, cannot be the same without allowing errors larger than $d\lambda = \cdot 05$. If they are to be the same the excessive error is probably in 21766, which is nebulous and would require $d\lambda = \cdot 08$. Further, in addition to the lines assigned here to the D series, there are a very large number of other lines showing separations of 788. The question arises, therefore, as to the origin of this abnormality, and it is important to discuss the various possible sources. The formula gives so closely the values of the lines for D_{11} from $m = 1$ to $m = 7$ that there can be little doubt as to the essential correctness of the D_{11} allocation. The limit of the series cannot then be very different from $S(\infty)$.

(1) Is 788 a real separation—*i.e.*, is it produced by a displacement on $S_1(\infty)$ by a larger oun multiple—in this case of $44\frac{1}{4}\delta$ in place of 44δ? If so the separation would be 4·52 greater, or 791 instead of 788, and such an explanation is therefore quite inadmissible.

(2) Is it a b link modified by displacement? If $D(\infty)$ be as found, *i.e.*, $(-\delta_1)S(\infty)$, ν_1 will be increased by ·09 or 786·45 to 786·54—an inappreciable change. To produce a change of 2 in the value of b or ν_1 the limit would have to be $(-5\delta)S_1(\infty)$, which gives a value 88·5 above $S(\infty)$. But, as a fact, the limit found is quite close to $S_1(\infty)$. This explanation is therefore excluded.

(3) Are these displacements on the d sequences? In the normal case the d sequences for a given satellite triplet are the same, but are displaced from one satellite set to another. Is it possible that the sequences suffer displacement in the same triplet also? Take, for example, the satellite set whose first line is 20669. The sequent is $d = 51655 - 20669 = 30986 \cdot 20 = N/(1 \cdot 881350)^2$. A displacement of $-\delta_1$ on the denominator 1·881 increases d by 2·06. If the displacement is on the first line of the triplet it must be $-\delta_1$, if on the second $+\delta_1$, and both give practically the same value 788·51 for the apparent separation in general agreement with the observed value. There is nothing, however, to show whether the displacement should be $-\delta_1$ on the d_1 or $+\delta_1$ on both d_2, d_3, as the observed 308·82 is within our assumed error limit, but it is interesting to note that if the oun for ν_2 be the same as for ν_1 the true value of ν_2 would be very close to the observed. In this connection it is to be

remembered that in the d sequences the oun is not affected by the peculiar triplet modification shown by all elements. In the 20842 set the observed ν_2 is 307·68, but the value of the observed $\nu_1+\nu_2$ is very nearly normal. This means that the third line does not suffer displacement, but only the middle one 21631.

If this explanation is correct, the modifications must diminish with increasing order. For instance, in $d(2)$, δ_1 produces a change in separation of ·56, and the new $\nu_1 = 786·45+·56 = 787·01$ as against 787·16 observed. For $m = 3$ δ_1 produces ·23, but the possible observation errors in n—maximum $dn = 1·0$—are now so great that the observed separation of 787·99 is well within the limits of 786·68. So far then as merely numerical agreement goes this explanation would seem very satisfactory, but the changes required are so small that by themselves they can give little confidence. We shall, however, see later how it explains certain effects in the F (∞) (p. 368)—which depend on the $d(1)$ sequents—and, further, how it also explains similar modifications in the other elements of this group. Meanwhile further evidence in its favour may be obtained from linkage considerations. Some examples follow.

(*Note.*—The observation errors in the separations should not exceed about ·50.)

(a) The mesh

	786·03	(1) 20714	**311·84**						
(1) 19928				(2) 21026	**787·36**	(4) 21813	**309·47**	(1) 22123
	789·67	(3n) 20718	**308·20**						

Here with 20714 ν_1 is normal, ν_2 abnormal, but $\nu_1+\nu_2 = 1097·87$. Our explanation gives $788·51+309·20 = 1097·71$. Thus on the upper set the first and second have the same d sequent, whilst the third has $(\delta_1)\,d$. In the lower set, on the other hand, 20718 has $(\delta_1)\,d$, the same as for the third.

So also in D_{18}. In the first set d has δ_1 in second and third lines, in second set it has $2\delta_1$ in second line.

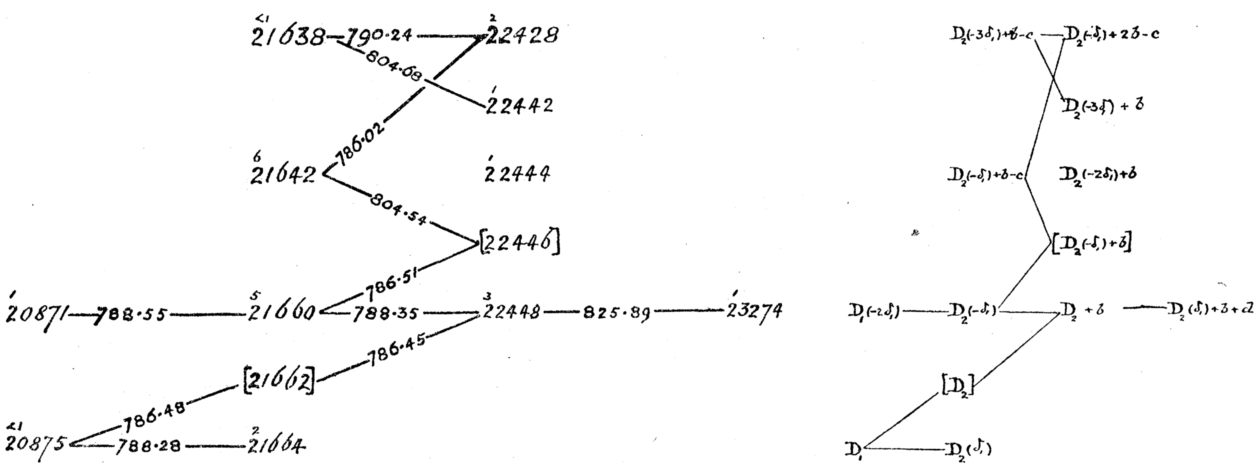
(b)

	786·43	(2) 23390	**788·77**			
				(3) 24178	**308·77**	
(4) 22603	**789·15**	(4) 23392	**786·03**			
	311·03	(1) 22914		**2 × 786·47**		(4) 24487

This is a striking example of persistence of the displacement in linked lines. Of the lines in the second column, the first has the same sequence term as 22603, the next two have the links ν_1, ν_2, but the sequent displaced δ_1. The line 24178 keeps the same displaced sequent as these last, and therefore has a normal ν_1 to 23392, but the abnormal to 23390. So 24487 keeps this same δ_1 displaced sequent and so shows a normal ν_2 to 24178 and $2\nu_1$ to 22914. In other words, if the first line is denoted by X, the above sets may be denoted by the following scheme :—

	$X+\nu_1$		
		$X(\delta_1)+2\nu_1$	
X	$X(\delta_1)+\nu_1$		
	$X(\delta_1)+\nu_2$		$X(\delta_1)+2\nu_1+\nu_2$

(c) This example involves the c, d links whose values are respectively 804·59, 823·29. It belongs to the D(1) system. The lines enclosed in [ ] are hypothetical and introduced to indicate the transitions. The relations are indicated by the accompanying schematic arrangement, starting with 20875 = D_1.



It may be noted that in this fragment of a linkage all the links involved are p-links. A similar preponderance of these in linkages connected with D lines is a marked feature also in Ag and Au [IV., p. 389].

The conclusion is to be drawn that there is very considerable evidence in favour of (3) as explaining the origin of these modified separations. It does not, of course, follow that the effect indicated in (1) does not exist amongst the lines of a spectrum. We know also that the effect indicated in (2) is existent, and we shall find clear evidence of it in the existence of lines depending on limits which are displaced S(∞), *e.g.*, in the case of 19928 considered below. Further support to (3) is given by the F separations (p. 370).

The Satellites.—Any allocation of satellites which may be regarded as firmly established is a matter of some difficulty on account of the large number of ν_1, ν_2 separations which enter indirectly as links, and the prevalence of sets depending on displaced limits and displaced d sequences. The lines in the list for $m = 1$ are placed there provisionally and for special discussion, although other lines certainly belong to the system. Consequently, the denotation D_{1n} is to be regarded simply as a means of referring to the different sets considered in this communication only, the n giving the ordinal position starting from 20976 as D_{11}.

The criteria that these lines should be possible D satellites with the same D_{11} are that the differences of their mantissæ from that of D_{11} should be multiples of the oun. Should any belong to another group with limit $(y\delta_1)$ D(∞), their mantissæ will be modified, but the qualification test will still hold with reference to the difference between their mantissæ thus modified and that of the original D_{11}, for they must all belong to d sequents and so be oun multiples. Nor will the test be affected by the existence of the supposed cause of the abnormal D triplet separations. The value of δ given by both the $S_3(1)$ has been seen to be 249·6 with $\Delta'_2 = 4243·2$, $\Delta_2 = 4780·0$. The maximum observation errors will be taken as ·03 A and the actual O−C error be written $-·03p$. The mantissæ for the various D_1 lines are then

1.	20976	$890749-30·82\xi+4p$,	
2.	20875	$887648-30·66\xi+4p$,	$3101 - ·16\xi+4(p_1-p_2) = 12\frac{1}{2}\delta-19-\ldots$
3.	20871	$887520-30·66\xi+4p$,	$3229 - ·16\xi+4(p_1-p_3) = 13\delta-16-\ldots$
4.	20842	$886640-30·61\xi+4p$,	$4109 - ·21\xi+4(p_1-p_4) = 16\frac{1}{2}\delta-9-\ldots$
5.	[20763]	$884207-30·50\xi+4p$,	$6542 - ·32\xi+4(p_1-p_5) = 26\frac{1}{4}\delta-10-\ldots$
6.	20669	$881350-30·36\xi+4p$,	$9399 - ·46\xi+4(p_1-p_6) = 37\frac{3}{4}\delta-23-\ldots$
7.	19928	$859253-29·30\xi+3p$,	$31496-1·52\xi+4(p_1-p_7) = 126\frac{1}{4}\delta-16-\ldots$
8.	19116	$835908-28·21\xi+3p$,	$54841-2·61\xi+4(p_1-p_8) = 219\frac{3}{4}\delta-8·6-\ldots$.

The test requires that the expressions to the right of the first term in the last column should vanish. The effect of any possible value of ξ is small. Further, as δ is probably known within $\pm·2$, it is only in the two lines, 7, 8, that it can be effective towards satisfying these conditions. It is clear then that the conditions can be satisfied within possible observation errors by Nos. 4, 5, 7, 8, and not by Nos. 2, 3, 6, nor by 8, if δ requires diminution by as much as ·1.

Considering 1, 4, 5, it may be noticed that the separations of 4, 5 from 1 are due to $16\frac{1}{2}\delta$ and $26\frac{1}{4}\delta$. Since $16\frac{1}{2}\times 8 = 132$ and $26\frac{1}{4}\times 5 = 131·25$, these separations are in the normal triplet satellite separation ratio. Moreover, the mantissa of the extreme satellite $884207 = 189\times 4678·3 = 189\Delta_2$ within our present degree of approximation to δ. This again satisfies the normal rule. It is clear, therefore, that these three lines form a normal triplet D (1) set. As to Nos. 7, 8, the ratio of separations is not that of two satellites to 20976 as D_{11}, although near it. The mantissa of (8), however, is $835908 = 197\times 4243·1 = 197.\Delta'_2$ within our present approximation to δ, and suggests that it is the extreme satellite of another group. Returning to the other lines it remains to see if they satisfy a test with a displaced D (∞). Now $y\delta_1$ on D (∞) produces a change of $-4·42y$ and consequent changes in the mantissæ as follows: $135·51y$ in (2), (3), $134·18y$ in (6), $124·7y$ in (8). As $124·80 = 2\delta_1$ it follows that the test for (8) is unaltered and those for 2, 3, 6 become

$$-10·71y-19-\ldots = 0; \qquad -10·71y-16-\ldots = 0; \qquad -9·38y-16-\ldots = 0;$$

whereas in (8) the corresponding condition is not changed, but the mantissa becomes $835924+2y\delta_1 = 197(4243·1+·63y)$. In this case a displacement of one oun in the limit produces the same effect as that of two ouns in the sequent. The test for the others is very closely satisfied by $y = -2$, or the supposition that they belong to a group with a limit $(-2\delta_1)\,D(\infty)$ *or* with a limit $(+4\delta_1)\,D(\infty)$, decreasing by δ the mantissæ of the sequents and requiring

$$\cdot 6 - \ldots = 0\,; \qquad 3\cdot 5 - \ldots = 0\,; \qquad 8\cdot 9 - \ldots\,;$$

which again are easily satisfied within observation errors. With regard, however, to (2), (3), it is not common to find two satellites so close together, and as they both belong to doublet sets we should suspect that they belong to different groups, and in fact the condition is satisfied by regarding the limit of 20875 as displaced by $-\delta_1$ on that of 20871.

We shall consider the conditions more fully later in connection with the more accurate determination of the oun. The preceding results are sufficient to show that (1), (4), (5) are a definite normal D satellite on the basis of $\Delta_2 = 4680 \pm$. That (7) (8) belong to a series probably based on $\Delta'_2 = 4243 \pm 1$, and 2, 3, 6 to a parallel system based on the limit $(-2\delta_1)\,D(\infty)$ or $(\delta)\,D(\infty)$.

I omit details about the satellite *e. u, v* links, but the following points are interesting. The line 19116 affords an example of the two displaced D_2 lines analogous to that given in 3(α) as explaining the source of the abnormal ν_1 separations. In addition the linked line $2e + D_{18}$ shows the same effect with two lines (2) 25380·26, (1) 25382·58. D_{16}, D_{36} are curious as showing successive abnormal u links. Thus

D_{16}

(1) 18780	**1888·61**	(2) 20669	**1888·37**	(4) 22557		

D_{36}

(1) 19881	**1885·31**	(3*n*) 21766	**1885·71**	(3) 23652·29	
					3·69
			1889·40	(3) 23655·98	
					3·31
				(1) 23659·29	

The two lines D_{12}, D_{13} which have been shown to have a relative displacement of $2\delta_1$ show u, v links to a mid-line displaced δ_1 from each. Thus (3) 22815·84 is 1942·15 or $v - \cdot 29$ below their mean; whilst (1) 19778·52 is 1883·69 or $u - \cdot 34$ below the mean of D_{22} and D_{23}.

The satellites for $m = 2$ are more doubtful on account of the large changes in the mantissæ produced by observation errors. The line 38531 has a mantissa difference from that of d_{11} of $12\frac{1}{2}\delta$, exactly the same as in D_{13} (1). It should, however, be one of a doublet with the second line stronger. The others show mantissæ differences of 28δ, 103δ, 124δ, $224\frac{3}{4}\delta$, which, allowing for the usual changes in the second order, clearly point to the sets as indicated in the table. There appears, however, no analogue in $m = 1$ for 38324. In this case it is important to notice that the ν_2

separation 308·99 corresponds to the displaced δ_1 sequent for $m = 2$. It thus supports the explanation adopted for the triplet modification.

For reference the multiples of the oun giving the satellite separations are collected in the following table, where d_1, d_2 refer to the first and second line in a satellite set.

$m = 1$.			$m = 2$.		
n.	d_1.	d_2,	n.	d_1.	d_2.
			3	12½	
2	12	12¼	5	28	28¼
3	12½	12¾	?	103	103¼
4	16½	16¾	7	124	
5	26¼	26½	8	224¾	225
6	37¾	38			
7	126¼	126¼		$m = 3$.	
8	219¾	220			
			4	19½	

KrF.—The F lines form parallel series in which the constant separations depend on the satellite separations of the D series. In other words the limits are the $d(1)$ sequents which form the satellites. Now it has been shown above that the abnormal triplet separations which the D series exhibit is probably due to the fact that the d sequent for a given satellite triplet is not the same for each of the three lines, but that they are subject to a displacement of one or more ouns. For instance where ν_1 is 788 in place of 786 (in round numbers) the difference 2 is due to the fact that d_{2n} is not equal to d_{1n} but is $d_{1n}(\delta_1)$. If the strongest line is to be taken as normal, we should expect the d_{3n}, or d_{2n} to be normal rather than d_{1n} as D_{1n} is always weaker than the other lines of a triplet satellite. In this case $F_n(\infty) = d_{1n}(\delta_1)$ and the F separation $= F_n(\infty) - F_1(\infty) = d_{1n}(\delta_1) - d_{11}$ which is less than the observed satellite separation by about 2. As a fact we do find these diminished separations. In searching for F lines therefore we have to examine the spectrum for wave-lengths longer than d_{11} and showing as multiplets with separations the same as the satellite separations or less. In the particular case of Kr these are 100, 105, 133, 213, 307, 1048, 1860,* and we are to expect series which we will denote by F_n with n from 1 to 8. From this point of view it is unfortunate that allowance has to be made for the rule as to the excessive displacements occurring in the lower orders ($m = 2, 3$) and that a complete multiplet, showing all the above separations, is not to be expected. With $D_{11}(1) = 20976·63$ and $D_1(\infty) = 51655·56 + \xi$, $d_{11} = 30678·93 + \xi = F_1(\infty)$. The mantissa of f is in general large, say, beween ·7 and ·99. Consequently the region in which $F_1(2)$ is to be found is where the wave-number is less than $30768 - N/(2·99)^2$,

* There may, of course, be others depending on D lines other than those considered in the text.

or say, <18491, or $\lambda > 5408$. This region is examined for the separations in question and lists made. It is then found that some depend on the same first line, in which case, they clearly refer to F or related lines. Several sets are found connected by one of the ordinary links, which excludes at least one of them as a direct F line. It is now easy to select a few sets from the lists which seem suitable for the F_1 line. This with the given $F_1\infty$ gives $f(2)$, and then RYDBERG'S tables give a rough approximation to F (3). It is then only necessary to examine the lines near these for the separations, in order to find the actual $F_1(3)$. The result of this examination is to show that for the first three orders $m = 2, 3, 4$, the only sets which exist without displacements in any correspond to the separation 301, with F_1 lines (1) 17321·51, (1n) 23353·84, (3) 26057·20. The formula calculated from these gives as the limit 30678·64. This is only ·29 less than d_{11} and is thus in very satisfactory agreement with the rule. Using the value of d_{11} as found from the D series with $D(\infty) = 51655{\cdot}56+\xi'$ the actual formula is given by

$$n = 30678{\cdot}93+\xi'-N\Big/\left\{m+{\cdot}877406-577{\cdot}8\xi'-\frac{{\cdot}023916-363{\cdot}3\xi'}{m}\right\}^2.$$

In this if $D(\infty) = S(\infty)$, $\xi' = \xi-4{\cdot}27$, but if $D(\infty) = (-\delta_1)\,S(\infty)$, $\xi' = \xi+{\cdot}15$.

The mantissa of the first line $F_1(2)$ is

$$865448-107{\cdot}26\xi+16p = 185\,(4678{\cdot}1-{\cdot}580\xi'+{\cdot}034p) = 185\Delta_2{}^*$$

within error limits. The lines have been selected as showing the given separations. Quite independently they give a formula with the proper limit and with the first sequence mantissa a multiple of Δ_2. The evidence therefore for the correctness of the allocation is incontrovertible.

In the consideration of the notation for the various parallel F series it will be necessary to determine what is to be understood by the normal separations. If the latter are to be decided directly from the $D_{11}(1)$ lines, the separations must be those given above. But a glance at the table of F lines will show that there is a considerable variation from these, and indeed from one another—due as we have seen to the great variability in the oun displacements. Especially is this noticeable in the separations given by the $D_1(1)$ satellites as 213·38 and 307·27. In place of these we find values about 2, 4 or 6 less, corresponding to 1, 2 and 3 oun displacements. The separation 301 is the most frequent displacement from 307·27. Now δ_1 alters the separation by 2·03, and therefore $3\delta_1$ alters 307·27 to 301·18. That these deviations from normal values correspond to real F separations can be seen by their frequent repetition in connection with F lines. See for instance the maps for F, especially F (5), in Plate 4.

* $185\Delta_2 = 204\Delta'_2+3\delta_1$.

KrF.

[For brevity F (*m*) is printed as F in each order.]

$m = 2.$

(1) 17321·51		F_1	(5) 17594·17		$F_1 (7\Delta'_2)$
(1) 17622·73	**301·22**	$(3\delta_1) F_6$	(1) 17695·04	**100·87**	$F_2 (7\Delta'_2)$
[20504·86]	e	$F_1.e$	(5) 20991·16*	**213·64** + e	$F_5 (7\Delta'_2).e$
(<1) 20813·91	**309·05** + e	$(-\delta_1) F_6.e$			
			(1) 19134·54		$(-2\delta_1) F_1 (51\Delta'_2)$
(6) 17747·14		$F_1 (10\Delta_2)$	(1) 19343·33	**208·89**	$F_5 (51\Delta'_2)$
(2) 17952·16	**205·02**	$(\delta) F_5 (10\Delta_2)$	(1) 19348·50	**213·96**	$(-2\delta_1) F_5 (51\Delta'_2)$
			(5) 20991·16*	**1856·62**	$F_8 (51\Delta'_2)$
(2) 17972·78		$F_1 (16\Delta'_2 + \Delta_2) \equiv X_1$	[22317·89]	e	
(1) 18108·55	**135·77**	$(-\delta_1) X_4$	(3) 22448·65	**130·76** + e	$F_4.e$
(1) 18177·75	**204·97**	$(3\delta_1) X_5$	(4) 22531·66	**213·77** + e	$(-2\delta_1) F_5.e$
(2*n*) 18282·20	**309·42**	$(-\delta_1) X_6$	(3) 24178·77	**1860·88** + e	$(-2\delta_1) F_8.e$
[21156·13]	e				
(1) 21258·41	**102·28** + e	$X_{23}.e$			
(1) 22203·47	**1047·34** + e	$X_7 (-2\delta_1).e$			
(<1) 23013·63	**1857·50** + e	$(\delta_1) X_8.e$			

$m = 3.$

(1*n*) 23353·84		F_1	(3) 23418·38		$F_1 (4\Delta'_2)$
(1) 23659·29	**305·45**	$(d_1) F_6$	(4) 23518·73*	**100·35**	$F_2 (4\Delta'_2)$
(4) 25213·56	**1860·72**	F_8	(5) 25278·06	**1059·68**	$F_8 (4\Delta'_2)$
(5) 21573·49	**103·68** − u	$u.F_3$			
(7) 24390·12	**213·00** + d	$F_5.d$	(2) 23507·07		$F_1 (10\Delta'_2 + \delta)$
(4) 23518·73*	**1048·92** − u	$u.F_7$	(1) 23639·82	**132·75**	$F_4 (10\Delta'_2 + \delta)$
(2) 23340·79		$(6\delta_1) F_1 (-\delta)$	(1*n*) 25367·06	**1860·00**	$F_8 (10\Delta'_2 + \delta)$
(1) 23554·41	**213·62**	$(6\delta_1) F_5 (-\delta)$			
(<1) 23349·04		$(2\delta_1) F_1$			
(3) 23655·98	**306·94**	$(2\delta_1) F_6$			

	$m = 4.$			$m = 5.$	
(3) 26057·20		F_1	[27497·70]		F_1
(1*n*) 26157·54	**100·34**	F_2	(9) 27708·28	**210·37**	$(2\delta_1) F_5$
(3) 26189·73	**132·53**	F_4	(7) 29357·89	**1860·19**	F_8
(4) 26358·25	**301·05**	$(3\delta_1) F_6$		———	
(1) 26265·05		$F_5 - 2x$	(1) 26727·89		$a.F_1$
[26270·16]	**212·96**	F_5	(7) 26863·38	**135·49**	$a.F_4$
(4) 26275·27		$F_5 + 2x$			
(1) 26065·15	**16·05**	$F_2 + 3x$	(4) 27482·72	− **15**	F'_1
	———			———	
(2*n*) 26067·66		$F_1 + 4x$	(4) 27588·11	**105·39**	F'_3
(2*n*) 26369·36	**304·21**	$(2\delta_1) F_6 + 4x$	(1) 27784·33	**301·61**	$(3\delta_1) F'_6$
(1) 27924·30	**1859·15**	$F_8 + 3x$	(6) 28535·88	**1053·16**	$(-2\delta_1) F'_7$

See also Map F (5).

* At least one a coincidence.

KrF (continued).

$m = 6$.

[28356·47]		[F_1]
(1) 28664·88	308·41	F_6
(2) 26779·99	**307·71 − u**	$u.F_6$
(4) 30218·69	**1862·22**	$(-\delta_1) F_8$
(1) 28361·39	**4**	$(-2\delta_1) F_1$
	———	
(1n) 28574·61	**213·22**	$(-2\delta_1) F_5$
(10) 27525·16	**1047·96 − u**	$u.(-2\delta_1) F_7$
	———	
(1) 28340·49	**−16**	F'_1
(3) 28444·08	**103·59**	F'_3

$m = 7$.

[28907·8] = u.30791·40		[F_1]
(7n) 29005·44	**97·62**	$(\delta_1) F_2$
(2) 30791·40	**u**	$F_1.u$
(3) 30999·10	**207·70 + u**	$F_5.u$
(3) 31841·03	**1049·65 + u**	$F_7.u$
(6) 28891·63	**−16**	F'_1
	———	
(3) 29198·53	**306·90**	F'_6
(1) 30875·06	**99·24 + u**	$F'_2.u$
(1) 31081·96	**306·14 + u**	$F'_6.u$
(6) 31823·11	**1047·29 + u**	$F'_7.u$
(5) 32635·69	**1859·87 + u**	$F'_8.u$

$m = 8$.

[29286·33]		[F_1]
(3) 29498·44	**212·11**	F_5
(7) 28808·81	**309 − b**	$b.F_6$

$m = 9$.

[29552·20]		[F_1]
(2) 29857·70*	**305·50**	F_6

$m = 10$.

[29728·00]		[F_1]
(6) 29823·86	**95·86**	$(2\delta_1) F_2$
(1n) 30036·53	**308·53**	F_6
(2) 29857·70†	**129·70**	$(2\delta_1) F_4$
(1n) 30777·90‡	**1049·94**	$(-\delta_1) F_7$

[29712·00]	[F'_1]
(3n) 29845·84	F'_4
(2n) 29926·23	F'_5
(1n) 30022·82	$(-\delta_1) F'_6$

* Or $(2\delta_1) F_4$ (10).

† Or F_6 (9).

‡ Is probably $(-\delta_1) F'_1 (7) u$. It is very diffuse and may be both.

It will be most convenient to deal with the multiplets order by order. We are to expect great displacements in the f sequences in the first two orders—single pairs displaced bodily—if the analogy with the triplet systems of the alkaline earths holds in the rare gases. Also throughout the orders we shall find not only in Kr but in the other gases small oun displacements in the limits, owing to the instability of the d sequences.

$m = 2$. In the first order there appear only one normal pair F_1 (2), F_6 (2); but ν_5, ν_6, ν_7, occur in a parallel set linked to the normal by the e link.

The following sets occur amongst others:—

(1)

			1049·06	(2) 18744		
(5) 17594	**100·87**	(1) 17695;	**98·37**	(1) 17692·54		
			307·84	(1n) 18002	**97·85**	(2) 18100

Here there is an example of ν_2 or of ν_6, ν_7 appearing as links, omitting 97·85 as a doubtful connection. The ν_2 is the exact D satellite separation. Moreover, the mantissa of 17594 is $895149-110{\cdot}63\xi$, differing from that of $f(2) = 865448-107{\cdot}26\xi$ by $29701-3{\cdot}37\xi+16{\cdot}5p = 7(4243{\cdot}00-{\cdot}48\xi+2{\cdot}3p) = 7\Delta'_2$. This is a clear displacement by $7\Delta'_2$, for ξ cannot be more than a few units and p is a fraction. It is clear, therefore, that 17594 is $F_1(2)(7\Delta'_2)$ and 17695 is $F_2(2)(7\Delta'_2)$. The next two must therefore be only the linked lines $F_2(2)(7\Delta'_2)+\nu_6$, $+\nu_7$. There is a line (1) 17692·54, separated by 2·5 from 17695, which might suggest the displacement δ_1 in the limit. But 2·5 is too large, and the line itself is really a linked line to D_{12}, viz., $(e)\,D_{12}(1)$.

(2)

(2) 17972·78 **135·77** (1) 18108·55 ; **204·97** (1) 18177·75 ; **309·42** (2*n*) 18282·20

e

[21154·13] **102·28** (1) 21258·41 ; **1047·34** (1) 22203·47 ; **1857·50** (<1) 23013·63

This is a specially interesting displaced set in that it contains all the seven separations, three directly depending on the line 17972 and the remainder on a line linked to it by the *e* link. The mantissa of 17972 is $937966-115{\cdot}6\xi$, which is $72518-8{\cdot}35\xi$ above that of $f(2)$. This is very close to $16\Delta'_2+\Delta_2 = 72553$. The line may be written $F_1(2)(16\Delta'_2+\Delta_2)$. Call this X_1. Then remembering that δ_1 produces 2·03 in the limit and ·54 in the sequent, we see at once that in the others, for $135{\cdot}77 = 133{\cdot}74+2{\cdot}03$ is $(-\delta_1)\,X_4$ exact; for $204{\cdot}97 = \nu_5-8$ is $(4\delta_1)\,X_5$; for $309{\cdot}42 = \nu_6+2$ is $(-\delta_1)\,X_6$. The others depend on $e+X_1$. The separation 102·28 is the mean of ν_2, ν_3 which depend on two d sequents differing by $2\delta_1$. It may therefore be written as either $(\delta_1)\,d_{13}$ or $(-\delta_1)\,d_{12}$, we will write it $X_{23}+e$. For $1047{\cdot}34 = \nu_7-1$ the displacement is $-2\delta_1$ in the sequent, or the line is $X_7(-2\delta_1).e$. For $1857{\cdot}50 = \nu_8-2{\cdot}6$ and the line is $(\delta_1)\,X_8.e$.

(3)

			(1) 19343·33
(1) 19134·54		**213·96**	(1) 19348·50
	822·11	**1856·62**	(5) 20991·16
(1) 19956·65		**308·13**	(1) 20264·78

With $30678+\xi$ the mantissa of 19134 is $216805-133{\cdot}50\xi$ above that of $f(2)$. It is in the neighbourhood of $51\Delta'_2$. The whole set of lines are representable as follows (putting as before $\xi = -1{\cdot}34$) :—

$$
\begin{aligned}
19134 &= (-2\delta_1)\,F_1(2)(51\Delta'_2-2\delta_1) &&= (-2\delta_1)\,Y_1(-2\delta_1)\\
19348 &= (-2\delta_1)\,F_5(2)(51\Delta'_2) &&= (-2\delta_1)\,Y_5\\
19343 &= F_5(2)(51\Delta'_2) &&= Y_5\\
20991 &= F_8(2)(51\Delta'_2) &&= Y_8\\
19956 &= (-2\delta_1)\,F_1(2)(51\Delta'_2-2\delta_1)+d &&= (-2\delta_1)\,Y_1(-2\delta_1)\,.\,d\\
20264 &= (-2\delta_1)\,F_6(2)(51\Delta'_2-2\delta_1)+d &&= (-2\delta_1)\,Y_6(-2\delta_1)\,.\,d
\end{aligned}
$$

The numerical agreement is very close, and 19956 is a series inequality in which a change of 1 makes the two separations 822, 308 exact d and ν_6 links. On the other hand, as we have already pointed out, 20991 may be a D_{11} line depending on the limit $(7\delta_1)\,D(\infty)$.

The following points should be noticed in the foregoing allocations :—

(1) The presence of the large displacements in the sequence term by quantities differing from multiples of Δ_2 by one or two ouns—in this respect quite analogous to a corresponding effect in the alkaline earths. It seems to point to a kind of satellite effect in the F series analogous to that shown in the D_1, where the main strong line is displaced from the normal satellite depending on a multiple of Δ_2 by the addition of a few ouns. In this case the F satellite is in general too weak to be observed, except possibly the linked line $22203 = X_7(-2\delta_1)\,.\,e$ depending on $f(2)(17\Delta'_2)$.

(2) That where a multiplet line is absent, it frequently appears as a linked e line, but that the linked line never appears directly linked to the F_1.

(3) The line 20991 occurs twice as $F_5(7\Delta'_2).e$ and as $F_8(51\Delta'_2)$, also its possible existence as a kind of independent D_{11} line has already been referred to. It cannot of course be all, and at least two of the suppositions must be due to chance. It is also probable that such coincidences may occur in some of the other allocations. The evidence for the general effect is cumulative and not dependent on a single numerical agreement.

The F system of the first order ($m = 2$) have been considered in rather considerable detail in order to establish what appears to be a very general rule that in many groups of elements the configurations producing the normal F lines appear to have been subjected to a sort of explosive effect whereby other configurations producing f sequents displaced by large multiples of Δ_2 are produced. As a natural result the intensities of the normal lines in the spectrum are diminished since the observed intensities must depend on the number of emitting centres as well as the energy emitted by each. We have seen that they are displaced in pairs or sets containing one displaced F_1 line, but no attempt has been made to search for sets not containing the F_1. As we shall see later these displaced sets in the lowest order give a means of obtaining very accurate data for the determination of the value of the oun. In dealing with subsequent orders such a detailed discussion is not called for for this purpose.

$m = 3$. The normal lines are observed for F_1, F_6, F_8, and as illustrating the correctness of the explanation given above for the diminished separation 301 in place of 307 it will be noticed that the normal D value is shown by the line 23349. The other F lines of the set do not directly appear, but as in the case of $m = 2$, they are in evidence as linked lines. Some of the lines linked to $F_1(3)$, are represented in a map in Plate 5. The denotations of the lines are entered in place of the wave-numbers, which can be reproduced by adding the given separations, and each can be referred to by the column and order in the column in which it occurs. Again we have several examples where small changes (errors or displacements) in lines make all the allied links take their practically exact values, and so give evidence for the reality of each. Thus, $-1·2$ on **a**1 give c, ν_8; $-1·5$ on **a**2 give u, ν_7 and the mean of ν_2, ν_3. (or the line $(-\delta_1)\,F_2$ or $(\delta_1)\,F_3 = F_{23}$, say); $-·5$ ($d\lambda = ·08$) on **c**4* gives c, ν_8; $-1·5$ on **c**5 gives d, ν_4, ν_8. These new sets give us representatives of all the lines missing from the direct normal lines. It is noticeable how the f_8 sequent persists.

Certain displaced sets are given in the tables. If 23418 is ·5 less ($d\lambda = ·08$) the ν_2, ν_8 become exact and it is $F_1\,(3)\,(4\Delta'_2)$. The numerical proofs of these allocations are not given, as these displacements have no importance at present beyond the fact that they exist.

$m = 4$. Direct lines are found for F_n, $n = 1, 2, 4, 6$, whilst 5 appears displaced $\pm 5·01 = 2 \times 2·50$, to observed lines 26265, 26275. There is also a line 26067 ahead of 26065 by 2·51. If this 2·50 be due to some displacement it is probably $2\delta_1$ on the limit and some ouns on the sequence, or all by ouns on the sequence. The order is so high ($m = 4$) that it is not possible to decide, and it is shown in the tables as a difference $x = 2·50$. Linked lines are shown in the map (Plate 5). Again note that 2·5 on F_1 makes the v, u links exact, and that here again the x appears. Symmetry would seem to indicate that the true $F_1\,(4)$ or 26057 should be about x less. This would diminish the calculated limit of the series to a value nearer that given by the calculated $S\,(\infty)$.

For $m = 5 \ldots 10$ the values for F_1 calculated from the formula are 27498·81, 28357·47, 28909·97, 29286·33, 29554·21, 29730·00. With $\xi = -1·34$ as determined later, we should expect values less than these from about -1 for the first varying to -2 for the last. None of these appear but they have linked lines whilst other of the parallel F sets also appear directly.

$m = 5$. No line has been observed at 27498, but there are lines with it for F_5, F_8, and others for F_{137} by a link$-a$. A value of F_1 27497 is $F_8 - 1860·19$ and reduces it 1·1 as just suggested.† The connections are exhibited in the map (Plate 4). From this order and beyond there appears to be a parallel set at a distance 16 units less. For $m = 5$, this starts from 27482·72 as F_1. As is seen in the map (**c**9) it has a very large linkage to F lines with similar sets to those connected with the calculated F_1. We may explain its source as a displaced $(2\delta)\,F\,(\infty)$, as the difference of two p-links, $b-c$, or as the direct congeries of F lines depending on 20991 as an independent D_{11} line. In the map the notation depending on the second is adopted. In the list 27482 is written as F'_1 leaving the question of the origin open. The p-links are particularly prevalent. This was found to be the case also in Ag and Au, the only elements in which the linkages have been examined with any thoroughness. In particular the series of successive + and − links from 27497 recalls a similar

* This has been given as a bad $D_{11}.e$. The suggested change makes the e link worse, which increases the improbability of its belonging to the D system.

† The calculated is retained however in the map, as the links show the repetitions more clearly.

arrangement in the AgD (4) linkage shown in the c, d, e columns of the map for AgPiii [IV.]. The series is in fact continued further than is shown in the present map. Starting from 26727 we find $a-c+b-d+a-c+a-c$ (and $+d$) $= 3a+b-(3c+d)$, the actual separations being $770{\cdot}92-803{\cdot}607+787{\cdot}51-821{\cdot}81+771{\cdot}07-803{\cdot}66+770{\cdot}68-803{\cdot}16$ (and $+825{\cdot}35$). Further, it should be noted that each successive pair is a parallel inequality, one in excess and the other in deficit of normal value. It means an increased displacement $2\delta_1$ in each alternate line. But if the observation errors are small, there appear to be indications of simultaneous displacements in the f sequences as well as on the limit. In fact a similar phenomenon is indicated in the two next orders though naturally some elements are wanting. A precisely similar connection is shown by AgS (3), [IV., p. 382], in a still more striking and regular series of changes. The elucidation of the laws governing displacements is of the first importance and should be one of the immediate objects of investigation. For this purpose examples of continuous series of simultaneous and like displacements will be of the utmost value. For this reason maps of certain near lines (Plates 4, 5), are given for all the orders from 3 up to $m = 8$, but no attempt has been made to indicate exact displacements involving unity. The parallel series F′ about 16 below F exist for $m = 5, 6, 7$. The sets connected with F′ (7) all show the displacement unity. In the lists the true lines are entered as 1 less for $m = 6$ and 2 less for 7, 8, 9, 10 (*i.e.*, ξ about -2) than the calculated values. As is seen it makes the observed separations more normal and in so far supports the putting of the limit about 2 less. Later the actual change in the limit is found to be $-1{\cdot}34$.

Kr**F**. During the work of examining the X spectrum a new type of series, associated with the F series, came to light. Whilst the known F type depends on the differences of two sequences $d(1)-f(m)$, the new type has a series of lines whose frequencies are given by $d(1)+f(m)$. We shall denote the lines of these series by **F**, so that F will denote a difference frequency and **F** a summation.

We have already referred to the general properties of these series in the introduction. Some of the material from the Kr spectrum bearing on the subject are here collected. In the following lists each order is considered by itself. The examination has not been exhaustive so as to involve displaced values, but it is believed all the direct observed lines have been included. A few abnormal ones, with considerable displacement in the f sequence, have also been entered, as they raised questions which require future investigation. The F and **F** lines are arranged in parallel columns. The mean of the two corresponding lines is entered in thick type between them. That for the first corresponds to the fundamental limit. The succeeding ones are given in the form mean of the first + difference, and the difference only (which settles the denomination of the set) is entered. Thus for $m = 2$ the first mean is $30674{\cdot}77$, that for F_6, $\mathbf{F}_6$ is $30976{\cdot}55 = 30674{\cdot}77+301{\cdot}78$ and $301{\cdot}78$ is entered. Also over each line the difference from F_1 or $\mathbf{F}_1$ is entered. Notes on detail are appended below the lists. The evidence is clear as to the existence of a series of the form $A+f(m)$. If the limit were the same for both we should expect the mean to be 30678·93 + ξ instead of 30674·77. The latter is what should be expected if D (∞) = S (∞).

TABLE of F and **F** Lines.

	F.		**F.**	F.		**F.**
		$m = 2$.			$m = 5$.	
1.	17321	**30674·77**	(2) 44028·04 = ($3\delta_1$) **F**?	[27498]	**30678·23**	(4) 33857·67
	53		**148·79**			
2. 3.	17376	+ **102·10**	(1) 44176·83			
			133·77			
4.			(2) 44161·81			
	213·27		**209·68**			**214·23**
5.	*e*.20718	+ **211·58**	(8) 43433·13 *c*			(1) 34071·90
	301·22		**302·34**			
6.	17622	+ **301·78**	(3) 44330·38			(2) 33838·09 = $\mathbf{F}'_1$
	1033·85		**1052·14**			
7.	18355	+ **1043·00**	(1) 45080·18			(1) 34146·82 = $\mathbf{F}'_5$
	1872·65		**1844**			
8.	19194	+ **1858·40**	(1) 45872·18			

(for the displaced sets see p. 380)

		$m = 3$.			$m = 6$.	
1.	23353	**30677·27**	(2) 38000·71	[28357·47]	**30675·60**	[32993·80]
		$10\Delta'_2 + \delta$.				
1. 2.	23507	**30674·76**	[37842·45]			
			104·03			
3.			(1) 37946·48			
	132·75					**133·70**
4. 5.	23639					(2*n*) 33127·50
			306·41	**307·41**		**303·46**
6.			(4) 38148·86	28664		(1) 33297·26
			1048·54			
7.			(1) 38890·99			
	1860·00		**1859·62**	**1860·22**		
8.	25367	+ **1859·81**	(1) 39702·07	30218		

		$m = 4$ (F′).			$m = 7$.	
1.	26067	**30677·16**	(6) 35286·68	28907	**30674·71**	32441·62 = *e*.35624·97 33442·54 = *b*.33228·89
2.						
				F′.		
3.				28891·63		
			129·78			
4.			(1) 35416·16			
			207·17			
5.			(6) 35493·85			
	301·70					
6.	26369					
			1043·80			**1048·78**
7.			(1*n*) 36330·48	[F′ (7)]	**30677 34**	(1) 33511·28
	1856·6		**1859·18**			
8.	27924	+ **1857·92**	(1) 37145·86			

$m = 2$. To the only two direct F there appear two direct **F** with the same mean 30674·77 and modified separation 301·8. Also there is a direct line to $\mathbf{F}_4$. To the linked $\mathrm{F}_4.e$ corresponds a linked $c.\mathbf{F}_4$. In this connection it must be remembered that all the **F** are large, over 44000, and an e link would reach to lines outside the region of observation. Three other sets are included in the list, which involve displaced f sequents. The second pair give a mean 30674·77 + 102·10 and belong therefore to the limit midway between F_2 and F_3. But the sequent = half the difference = 13399·95 in place of 13353·26, on the supposition of a common limit. So also the pairs for $\mathrm{F}_{7,8}$ show f sequents 13362·41 and 13339·01 on the same supposition.

If the observed 44028 is really $(3\delta_1)\mathbf{F}_1$, where $\mathbf{F}_1$ has the same limit as F_1, and may be called the normal F_1, the mean of the observed F_1 and of $\mathbf{F}_1$ is 30677·80, and should give the true value of the limit subject only to observation errors on the two lines, *i.e.*, within maximum error of 1·1 with $d\lambda = \pm \cdot 05$, and within a probable error much less.

$m = 3$. $\mathbf{F}_1$ corresponds to F_1 with mean 30677·27 ± 1. There are some cases of displaced $f(3)$ as in $m = 2$. The $10\Delta'_2$ set appear also in **F**, and as they contain several examples they are placed in the list. There are two lines 37836·36 and 38050·15 (separation 213·79) which as F_1 and $\mathbf{F}_5$ give a mean 30671·71. The lines in the list show an unobserved line for $\mathbf{F}_1$, which is the basis for the others, its actual value is taken as $-3\delta_1$ displacement on 37836. The mean is 30674·76 which, on the supposition adopted above, corresponds to $a(3\delta_1)\mathbf{F}(\infty)$. Since $\mathrm{F}_3 = (-2\delta_1)\mathrm{F}_2$, the line 37946 may be written as normal $\mathbf{F}_2$, giving mean limit = 30676·79 ± 1. In the $4\Delta'_2$ set is a line 39799·89 = $\mathbf{F}_8(4\Delta'_2)$, giving with $\mathrm{F}_8(4\Delta'_2)$ a mean 1860·19 + 30678·78.

$m = 4$. There appear no direct **F** to the F lines. But they occur in the parallel set F′; but as $\mathbf{F}'_1$, $(2\delta_1)\mathbf{F}'_4$, $(3\delta_1)\mathbf{F}'_5$, $(2\delta_1)\mathbf{F}'_7$, $\mathbf{F}'_8$. The mean is 30677·16.

$m = 5$. Here $\mathbf{F}_1$ and $\mathbf{F}_5$ appear, but as the mean depends only on calculated F_1 it is not reliable. If F_1 be taken from the observed line 26727·89 by the $-a$ link, the F_1 line would be 27496·83, giving mean 30677·25. There are also lines connected with the parallel series F'_1 which has a limit 16 below F_1. F'_1 = 27482·72 and $\mathbf{F}'_1$ = 33838·09, gives mean 30660·40, which is about the proper amount below $\mathrm{F}(\infty)$ = 30677·80. With this goes 34146·82 as $\mathbf{F}'_5$ with separation 308·73.

$m = 6$. The unobserved lines supposed for the first pair are calculated respectively from the observed F_6, F_8, and $\mathbf{F}_4$. The line 33297 is $(2\delta_1)\mathbf{F}_6$. Corresponding to 33076·55 as $\mathbf{F}'_2$, the mean limit with F'_1 is 30658 in place of 30660. With this might possibly go 34018·93 as $(3\delta_1)\mathbf{F}'_7$.

$m = 7$. $\mathbf{F}_1 + e$ = 35624·97 gives a mean limit 30674·71. Also with $\mathbf{F}_1 + b$ = 33228·89 gives a mean limit 30675·12. Also 33511·22 an exact $\mathbf{F}'_7$ with mean 30677·34.

$m = 8$. $\mathbf{F}_1 + e$ = 35249·74 gives mean 30676·36, but F_1 is uncertain.

$m = 9$. I have not found $\mathbf{F}_1$, but 32015·37 as $\mathbf{F}_5$ gives $\mathbf{F}_1$ = 31802·00, which gives mean limit 30677·10.

$m = 10$. No $\mathbf{F}_1$ found, but 31726·38 as $\mathbf{F}_2$ and 31841·05 as $(-\delta_1)\mathbf{F}_5$ gives $\mathbf{F}_1$ the same value 31625·5. This gives a mean limit = 30676·75.

The evidence seems therefore clear for the existence of this type of series.

The Value of the Oun.—For the evaluation of the oun there are at disposal:—

(1) The Δ_1, Δ_2 as determined from the S separations. These have given (p. 346) for a first approximation to δ, the value 249·30 from ν_1 and 249·6 from the two alternative ν'_2, ν_2. The ν's are so ill-determined that these might possibly refer to values giving the same δ. But the fact that the value of e calculated from Δ_1 agrees so closely with the maximum ordinate in the corresponding occurrency curve (Plate 2, fig. 1) shows that Δ_1 must be exceedingly close to the true value, in which case it is

improbable that the ·3 difference in δ could be attributed to a single observation error on each of Δ'_2, Δ_2. As it has been shown in [III., p. 332] that the triplet separation always shows a slight difference in the δ from ν_1, ν_2, it is probable that the same occurs here also. The evidence there given goes to show that the value obtained from $\Delta_1+\Delta_2$ is always closer to the true value. We should expect, therefore, a value between the two values above.

(2) The evidence obtained from the D qualification test.

(3) The D satellites whose mantissæ depend on multiples of Δ_2, viz., 19116 on $197\Delta'_2$, and 20763 on $189\Delta_2$.

(4) The mantissa of $f(2) = 185\Delta_2$.

Before however conditions (3), (4) can be applied it is necessary to obtain if possible a closer approximation to δ from (2). The material for discussion is that given on (p. 366). We shall discuss it on the two bases of $\delta = 249{\cdot}60+x$ and $249{\cdot}30+x$ where x is certainly not greater than ·3. The complete conditions are, using the displaced values $(-2\delta_1)$ D (∞) for (2), (6) and omitting (3) as parallel to (2),

	$249{\cdot}60+x$	$249{\cdot}30+x$
(2)	$-2{\cdot}4+\ {\cdot}16\xi-4(p_1-p_2)+\ 13\tfrac{1}{2}x = 0$	$-\ 6{\cdot}5+\ldots = 0,$
(4)	$9{\cdot}0+\ {\cdot}21\xi-4(p_1-p_4)+\ 16\tfrac{1}{2}x = 0$	$4\ +\ldots = 0,$
(5)	$10{\cdot}0+\ {\cdot}32\xi-4(p_1-p_5)+\ 26\tfrac{1}{4}x = 0$	$2{\cdot}1+\ldots = 0,$
(6)	$-2{\cdot}7+\ {\cdot}46\xi-4(p_1-p_6)+\ 38\tfrac{3}{4}x = 0$	$-14{\cdot}3+\ldots = 0,$
(7)	$16\ +1{\cdot}52\xi-4(p_1-p_7)+126\tfrac{1}{4}x = 0$	$-21{\cdot}8+\ldots = 0,$
(8)	$8{\cdot}6+2{\cdot}61\xi-4(p_1-p_8)+219\tfrac{3}{4}x = 0$	$-56\ +\ldots+220x = 0.$

It is quite clear that the conditions in the first column cannot be satisfied without assuming very large observation errors unless x is negative, nor on the right hand column unless x is positive. In other words, δ must be $< 249{\cdot}60$ and $> 249{\cdot}30$. The first four equations, however, give no indications of amount, as the multiples of x are not sufficient to make the term in x more important than the error terms. In (6, 7, 8) the conditions may be written with $\xi \not> 1$.

(6)	$-\ 2{\cdot}7\pm\ 8{\cdot}5\ +\ 38\tfrac{1}{2}x = 0$	$-14{\cdot}3\ldots,$
(7)	$16\ \pm\ 9{\cdot}5\ +126\tfrac{1}{4}x = 0$	$-21{\cdot}8\ldots,$
(8)	$8{\cdot}6\pm10{\cdot}61+219\tfrac{3}{4}x = 0$	$-56\ \ldots.$

Nos. (6, 7) require x to be about equal and opposite in the two cases, say, $\delta = 249{\cdot}5$. This would make (8) give $-13{\cdot}5+2{\cdot}61\xi-4(p_1-p_8) = 0$. This last case offers some difficulties which we will consider later. For a further approximation we will therefore put $\Delta_2 = 4678+x$, $\Delta'_2 = 4241{\cdot}386+{\cdot}907x$ which give $\delta = 249{\cdot}4933+{\cdot}053x$, and $D_{15} = 20763{\cdot}25+dn$. Then, (p. 366)

$$d_{15} = 884207-30{\cdot}50\xi'+30{\cdot}50\,dn\pm{\cdot}5 \qquad = 189\{4678{\cdot}343-{\cdot}161\xi'+{\cdot}161dn\pm{\cdot}002\}$$

$$f(2) = 865448-107{\cdot}26\xi'+16\,p\pm{\cdot}5 \qquad = 185\{4678{\cdot}098-{\cdot}580\xi'+{\cdot}086p\pm{\cdot}002\}.$$

Hence

$$\cdot343 - x - \cdot161\xi' + \cdot161dn \pm \cdot002 = 0,$$
$$\cdot098 - x - \cdot580\xi' + \cdot086\,p \pm \cdot002 = 0.$$

Therefore

$$\left.\begin{aligned} \xi' &= -\cdot58 - \cdot38dn + \cdot20p \pm \cdot009 \\ x &= \cdot436 + \cdot222dn - \cdot032p \pm \cdot003 \end{aligned}\right\} \text{A.}$$

Here dn depends on an extrapolated value from a D_3 line, taken because the ν_2 showed a diminished value corresponding to the usual satellite modification of ν_1. If this is correct dn is the observation error on 21858 and is $\cdot14p$ with $d\lambda = \pm\cdot03$. This is certainly the most *à priori* probable supposition. If on the contrary hypothesis ν_1 is normal D_{11} is $21552\cdot65 + \cdot14p - 786\cdot45 = 20766\cdot20$ and $dn = 2\cdot95 + \cdot14p$. This gives

$$\xi = -1\cdot70 \;\; - \cdot05p + \cdot20p' \pm \cdot009,$$
$$x = \;\; 1\cdot091 + \cdot03p - \cdot03p' \pm \cdot003.$$

Taking now the case of $D_{18} = 19116$ we have already seen that a change of $y\delta_1$ on the limit, or $2y\delta_1$ is the sequent produce the same change, so that $(y\delta_1)\,D_{18}\,(2y\delta_1)$ is the same as D_{18}, and the D qualification test remains unchanged, although the mantissa is altered by $2y\delta_1 = 174\cdot8y$.

Supposing this displacement to take place the mantissa of D_{18} is

$$835908 + 124\cdot8y - 28\cdot21\xi + 3p \pm \cdot5 = 197\,\{4243\cdot188 + \cdot6330y - \cdot143\xi + \cdot015p_8 \pm \cdot002\}.$$

Here again, as in the case of D_{15}, the question arises whether the value of the normal sequent be taken as that of D_{18} or D_{28} which latter is $\delta_1 = 62\cdot4$ larger.

The condition then for the exact multiple of Δ'_2 is

$$1\cdot802 + \cdot633y - \cdot907x - \cdot143\xi + \cdot015p_8\,(\text{or} + 62\cdot4/197) = 0.$$

The two cases give

$$\left.\begin{aligned} \xi &= -\;\;\cdot58, \quad 1\cdot505 + \cdot6330y - \cdot020p_5 + \cdot03p' + \cdot015p_8 \quad (\text{or} + \cdot316) = 0 \\ \xi &= -1\cdot70, \quad 1\cdot063 + \cdot6330y - \cdot03\,p_5 + \cdot03p' + \cdot015p_8 \quad (\text{or} + \cdot316) = 0 \end{aligned}\right\} \text{B.}$$

Neither can be easily satisfied with d_{18}, but with d_{28} the conditions can be satisfied with $y_2 = -3, -2$, for the two cases respectively.*

The remainders in the two cases are $-\cdot077$ for $\xi = -\cdot58$ and $+\cdot113$ for $\xi = -1\cdot70$.

The list of F lines also give the following which can be used as tests.

(1) (6) $17747\cdot14 = F_1(2)\,(10\Delta_2)$, *i.e.*, $f(2)$ mantissa $= 195\,\Delta_2$,

(2) (5) $17594\cdot17 = F_1(2)\,(7\Delta'_2)$,

(3) (2) $17972\cdot78 = F_1(2)\,(16\Delta'_2 + \Delta_2)$.

* See, however, final order below.

The mantissa of (1), using $F(\infty) = 30678{\cdot}93+\xi'$ is

$$912207-112{\cdot}60\xi'+11p_1\pm{\cdot}5 = 195\{4677{\cdot}986-{\cdot}577\xi'+{\cdot}05p_1\pm{\cdot}002\}.$$

This requires $-{\cdot}014-x-{\cdot}577\xi+{\cdot}05p\pm{\cdot}002 = 0$, or combined with the condition for $f(2)$, $-{\cdot}112+{\cdot}003\xi-{\cdot}086p+{\cdot}05p_1\pm{\cdot}002 = 0$ and can be satisfied within error limits.

The mantissa of (2) $895149-110{\cdot}63\xi'+11p_2$ which differs from that of f (2) by $29701-3{\cdot}37\xi'+11p_2-16p' = 7\,(4243{\cdot}00-{\cdot}48\xi+1{\cdot}6p-2{\cdot}3p')$.

This requires $1{\cdot}61-{\cdot}907x-{\cdot}48\xi+1{\cdot}6p-2{\cdot}3p'=0$ easily satisfied for both cases within observation errors.

The mantissa of (3) is $937966-115{\cdot}6\xi'+11p_3$, differing from that of $f(2)$ by $72518-8{\cdot}35\xi+11p_3-16p$. With $\xi = -{\cdot}58$ and $-1{\cdot}70$ this becomes 72523·1... and 72532..., or $17\Delta'_2+1\tfrac{3}{4}\delta-23$... and $17\Delta'_2+1\tfrac{3}{4}\delta-24$... on their respective values of D'_2. The amount 23 is perhaps excessive to be covered by the various possible errors but it just comes within. It may be noted that $17\Delta'_2+1\tfrac{3}{4}\delta = 16\Delta'_2+\Delta_2$. These three data do not decisively distinguish between the two cases. This, however, is not to be unexpected because the two arise from a δ_1 displacement in d_{15}, the sequents in this neighbourhood are such that δ_1 on the limit and $2\delta_1$ on the sequent are nearly equivalent, and the multiples involved 185, 189, 195 are too close to produce contrasts. Incidentally, also, the discussion strengthens the allocation of the lines to the displacements given.

The only further test with our present knowledge is to obtain some independent evidence as to the exact value of the limit, and naturally we turn for this to the mean of the F and **F** series. The series however in Kr is not nearly so well developed as in X. As has been already seen there are only three sets of observed pairs (m = 2, 3, 4) and these give for $F(\infty)$ respectively values of 30674·77, ... 7·27, ... 7·16. Since a displacement of δ_1 produces a change of 2·03 in $F(\infty)$ the first may be due to the fact that the line taken for **F** (2) is really $(3\delta_1)$ **F** (2), when the true mean would be 30677·81. It is natural to seek further as to the existence of summation lines corresponding to our last three examples. The result shows a most remarkable agreement. The sets are shown in the following list together with those obtained from the normal F and **F**.

m.	F.	$F(\infty)$.	**F**.	
2	(1) 17321·51	30677·82	(44034·13)	$(-3\delta_1)$ (2) 44028·04
3	(1n) 23353·84	30677·27	(2) 38000·71	
4	(2n) 26067·66	30677·16	(6) 35286·68	
$F_1(2)\,(7\Delta'_2)$	(5) 17594·17	30677·76	(1) 43761·38	$\mathbf{F}_1(2)\,(7\Delta'_2)$
$F_1(2)\,(10\Delta_2)$	(6) 17747·14	30677·73	(1) 43608·33	$\mathbf{F}_1(2)\,(10\Delta_2)$
$F_1(2)\,(16\Delta'_2+\Delta_2)$	(2) 17972·78	30677·70	(1) 43382·63	$\mathbf{F}_1(2)\,(16\Delta'_2+\Delta_2)$

These are remarkably concordant, especially when it is noted that the F (3, 4) are diffuse lines and not so susceptible of exact measurement as the others. The mean limit is 30677·72 and may be taken as practically correct. That calculated from the series, and used in the preceding discussion is $30678·93+\xi'$. This, therefore, requires a correction of $-1·21$. The equations A would be satisfied by $p = 1$, $dn = 2$, $\xi = -1·21$. As however this value of ξ is probably correct within ·1 the best value of Δ_2 is obtainable from $f(2)$, viz.,

$$\Delta_2 = 4678·098 + ·580\,(1·21 \pm ·1) + ·086 \times 1 \pm ·002$$
$$= 4678·80 \pm ·10$$
$$\Delta'_2 = 4242·18 \pm 06$$
$$\delta = 249·536 \pm ·005.$$

If the difference between δ as found from ν_1, ν_2 be real and depend on electronic changes as hinted at in the introduction, the changes calculate to 73·94 electrons = 74. In other words, the ν_2 would refer to mass of nucleus +37 electrons and ν_1 to mass of nucleus -37 electrons. Is it merely a curious coincidence that the atomic number of Kr is 38, that of H being 1 ; all the electrons acting in one way for ν_1 and in the opposite for ν_2? it being remembered that when S lines are emittted one electron at least is absent.

Xenon.— The X diffuse system appears to be a most complicated one. As we shall see later there appears to be a congery of series converging to limits which are collaterals of S (∞) = 51025, and connected with these there are again congeries of F series converging to limits collateral to the various d_{1n} sequents or, say, the normal F (∞). These F series further show the existence of satellites—in other words the f sequence is also subject to slight collateral displacements. This renders their disentanglement a very intricate problem not only in itself, but because it renders the region of the spectrum involved very crowded, with lines close together, with the consequence that coincidences occur which may not refer to real relationship. In fact there are cases where the calculated values of supposed lines of different series are the same within observation errors.* This crowding is also increased by the existence of the allied **F** series referred to above. The complete discussion of all these related series should afford valuable material for arriving at a knowledge of displacement laws. Here it will be sufficient to indicate the nature of the problem and to deal with the material so far as to give confidence in the results as to the assignment of series and especially as to accurate determinations of the oun and the various links.

As vacuum tube spectra approximate to the spark type, the difficulty of drawing definite conclusion from the existence of a triplet separation is again enormously increased by the presence of the link relations which these spectra show. In arc spectra the appearance of a ν_1 or ν_2 separation may always with some certainty be ascribed to the fact that the lines in question are directly connected with series terms.

* A case in point is $\mathbf{F}_1$ (5) and $\mathbf{F}_3$ (17) in the series next considered ; also F_3 (13) and $\mathbf{F}_2$ (19).

Here however no such certain conclusions can be drawn. They may enter as links. Although their true connection may ultimately be definitely settled the doubt as to whether they give true or false scents renders the task of unravelling most bewildering.

In about the region to be expected we find the set

(1) 19880·72 **1778·42** (5) 21659·14 **815·30** (10) 22474·44

showing from their separations and order of intensities an indubitable satellite set of D_3 type. Below these come a number of indubitable doublets of D_2 type, and then a number of strong lines of D_{11} type. There are also in this region an extremely large number of lines with separations between 1780—1785 corresponding to the enlarged links already found in the KrD spectrum. This portion of the spectrum is set out together with their separations in the following table:—

(2*n*) 17903·44	(3) 19632·44	**1780·84**	(3) 21413·28		
1772·81	(7) 19676·25	**1781·23**	(1) 21457·48		
	(1) 19785·84				
	(3) 19815·84				
	(1) 19829·47				
	(1) 19880·72	**1778·42**	(5) 21659·14	**815·30**	(10) 22474·44
	(1) 19942·53	**1775·45**	(10) 21717·98		
	(1) 19959·64	**1772·60**	(1*n*) 21732·24	**812·77**	(6) 22545·01
(3*n*) 18238·60	(1) 19989·72	**1780·27**	(6*n*) 21769·99		
1778·86	(1) 20017·46	**1783·71** }	(10) 21801·17		
1783·06	(1) 20021·66	**1779·51** }			
(7) 18266·88	(2) 20029·12				
1774·85	(2) 20041·73	**1782·55**	(1*n*) 21824·28		
		1784·54	(1*n*) 21865·47	**1784·76** }	(2) 23650·23
	(4) 20080·93				
		1786·02	(1*n*) 21866·95	**1783·28** }	
	(1) 20107				

(1) 20305·60		
(6) 20312·70	**1783·72**	(5) 22096·45
(4) 20320·25	**1776·17**	
(1) 20333·22	**1774·92**	(3*n*) 22108·14
(5) 20443·29		
(5) 20454·88		
(1) 20467·90		
(6) 20470·75		

{ (7) 20500·13	**1784·68**	(2) 22284·82
1777·11		
(9) 18723·02		
814·13		
(2*n*) 17908·89 }		
(3) 20529·92	**1780·35**	(7*n*) 22310·27
(8) 20559·08	**1785·64**	(<1*n*) 22344·72
(1) 20581·64		

Before however considering these lines in detail it will be desirable to take here a preliminary discussion which involves a new fact in series relationships, and at the same time will give some reliable data which have a bearing on the present problem of the actual D and F series. The line 20312 and those in its neighbourhood show the following sets of separations, viz. :—

1.	(1) 17615·06	**832·13**	(10) 18447·19	**1865·51**	(6) 20312·70				
2.	(6) 17638·55	**827·92**	(2) 18466·47	**1866·75**	(1) 20333·22				
		1864·74	(5) 19503·29	**829·93**					
3.	(3) 17628·29	**831·70**	(1) 18459·99	**1860·26**	(4) 20320·25	**1859·79**	(1) 22180·04		
4.	(1) 17772·38	**835·41**	(8) 18607·79	**1862·96**	(6) 20470·75				
5.	(3) 19503·29	**829·93**	(1) 20333·22	**830·62**	(2) 21163·84				
				1864·93	(2) 22198·15	**1863·97**	(1) 24062·12	**1862·95**	(<1) 25925·07

In the general survey at the commencement of the investigation a large number of separations of an amount near 1864 was noticed. Suspecting that it indicated the existence of a second type of sharp series, a second smaller separation (for triplets) was looked for, analogous to the $\nu_2 = 815$ of the 1778 set. A further separation was found for a value about 830.* The whole spectrum was searched within the limits 1864 ± 2. The result is shown in the occurrency curve of Plate 2, fig. 3. This curve is unique amongst those hitherto observed in its great height above the pure chance line and also in the steepness of its rise and culmination to a single definite peak. The search brought to light also a very considerable number of long successive chains and of meshes (see *e.g.*, Nos. 2 and 5 above) of the same amount, proving that 1864 enters not only on its original source, as due to a displacement on some fundamental sequence, but also as a link. Now the c link of the normal $\nu_1 = 1778$ is 1872·63 and $c(3\delta) = 1865{\cdot}16$, thus suggesting a possible origin, also a corresponding c link for ν_2, *i.e.*, a separation produced by a second Δ_2 displacement is 833·84, or with a modified $c'(3\delta) = 830{\cdot}70$. These suggest triplets formed by the same Δ_1, Δ_2 on $p(-\Delta_1)$ instead of on p, in other words, series whose limits are $51025+1778 = 52803$, 54675·6, 55509·4. But the way in which the separations enter with the suggested D line 20312 indicates that they stand in fundamental relation to it and neither in a linkage relation, nor with the limits named. For in the latter cases it would throw out of gear the whole relation of 20312 with the D set, which some provisional work had seemed to establish. In this work they were considered as part of the D system through 20312 regarded as a $D_1(1)$ line with 18447, 17615 as satellites. In this case the d_1 sequence is of the order $51025 - 20312 = 30713$. As against this idea is, of course, the greater intensity of 18447, the supposed satellite over that of the D_{11}, and also the fact that no normal ν_1

* As a fact, however, there are several others depending on oun multiples also present, but which at present we need not deal with.

separation occurs with 18447 or ν_1, ν_2 with 17615. But provisionally that was set aside for the moment. If they represented a special D set, the separations ought to reappear in a triplet series of the F type, and in the reverse order. From the sets already excerpted the lines (8) 18607·79, (10) 23915·72, (5) 26365·19 appeared to have all the signs (RYDBERG'S tables) of belonging to one series. The formula calculated from them brought to light a whole long series of observed lines. The limit found was 30724, close to the value already found (30713) as of the order of magnitude to be expected. This so far supported the supposition of the D relation, but there also came to light another result of evident importance in general theory—viz., the **F** series already referred to. The ordinary form of a series is one in which successive lines obey a formula of the type $A - \phi(m)$. In this case we find series associated with it whose successive lines are given by $A + \phi(m)$. This holds for each of the triplet sets, so that the complete series are given by $A \pm \phi m$, $B \pm \phi(m)$, $C \pm \phi(m)$, where $B = A + 1864$, $C = A + 1864 + 830$. Quite apart from the importance of this fact in the theory of spectral series the phenomenon is of special use in calculating the various constants on which the series depends. For instance the sum of the wave-numbers of two corresponding lines gives 2A, 2B, 2C, thus determining the values of the limits quite independently of the nature of the series formula used. Moreover, the displacements which so frequently occur in the F and D series in the sequence term introduces uncertainties. This happens in two ways. First through the modified ν_1 values in which it is not always possible to say whether the displacement is produced in the D_1 or the D_2 line. Secondly because the typical line in any order is often wanting and only appears with a very large displacement of multiples of Δ_2 on the sequence term. This effect, however, provided it occurs for both sets (F, **F**), does not influence the values of A, B, C thus determined. Cases in point are the Kr sets F (2) $(7\Delta'_2)$, F (2) $(10\Delta_2)$, $F_1(2)(16\Delta'_2 + \Delta_2)$ given on p. 380. In consequence it is possible to determine the separations $B - A$, $C - B$ independently of satellite or other displacements. That such sequence displacements occur in these 1864 series is shown by separations which deviate from the normal by more than observation errors.

But, further, the difference of two corresponding F and **F** lines, say $\mathbf{F}_1 - F_1$, $\mathbf{F}_2 - F_2$, $F_3 - \mathbf{F}_3$, should each give $2f(m)$, if as is the normal rule the sequence term is the same for each line of a triplet. When however—as we have seen in Kr, and shall find even more markedly in X—there are displacements in $f(m)$ for successive lines in a triplet, these differences will not be the same, and the observed separations will vary from the normal values. For instance, suppose $f(m)$ becomes $f(m) - x$ for the second set, and $f(m) - y$ for the third. The lines are $A \pm f(m)$, $B \pm (f(m) - x)\ldots$. The values of A, B, C calculated from the sums are not affected, and the real values of the separations given by $A - B$, $C - B$ are not affected although the observed separations are $\nu + x$, $\nu' + y$ and $\nu - x$, $\nu' - y$. In some cases we shall find evidence from close lines with different x or y—but the results are quite definite. If, however, in the corresponding terms of the F and **F** lines the f are different, then the value calculated from their sum shows a change from the normal limit. The effect shows itself at once and the interpretation is less certain. It is possible that where such an affect appears it may not be real, but due to the existence of the two close lines just referred to, of which one in each set is too faint to have been observed. Thus if the displacements are x_1, x_2 instead of finding $B-f+x_1$, $B-f+x_2$, $B+f-x_1$, $B+f-x_2$, the 2nd and 3rd, or the 1st and 4th may not have been observed and we should be led to a wrong conclusion by taking, say, the 1st and 4th as corresponding lines. There are cases of this kind and also where one only is absent—*i.e.*, we find one close doublet for one of the F or **F** lines.

The lines composing the series are given in the table below. The limit calculated from the first three F_1 lines was found to be $30724{\cdot}28 \pm 1{\cdot}80$, the uncertainty being due to supposed maximum observation errors of $\pm{\cdot}05$A in each line. The later discussion of the $\frac{1}{2}(F+\mathbf{F})$ rule will show that the limit should be very close to 30725·26 with an error probably $<{\cdot}3$. The formula was recalculated with this limit by supposing the three standard lines to be in error by $-{\cdot}02$, $+{\cdot}02$, $-{\cdot}02$, *i.e.*, by half their supposed maximum possible errors. The formulæ for the F_1 and $\mathbf{F}_1$ series then become

$$n = 30725{\cdot}26 \mp N \Big/ \left\{m + 1{\cdot}022746 - \frac{{\cdot}028705}{m}\right\}^2.$$

List of F and **F** Lines.

In each order the first line of numbers gives the F set, the second the **F** set. Between these are entered the mean values of the F and **F** which give the corresponding limits. When the values are deduced by methods explained in the notes they are enclosed in ( ), when calculated and not observed in [ ].

$m = 1$	(3010·35)	**1864·64**	(4875·09)	**830**	(5705·00)
	30725·77		**32590·35**		**33420·35**
	(58441·20)	**1864·50**	(60305·70)	**830**	(61135·70)
$m = 2$	(8) 18607·79	**1862·96**	(6) 20470·75	**829·46**	(<1) 21300·21
	30726·05		**32589·09**		
	(4) 31102·16.*e.v**	**1863·17**	(<1) 32965·33.*e.v*		
$m = 3$	(10) 23915·72	**1863·92**	(8) 25779·64	**829·69**	(1) 26609·33
	30725·15		**32589·28**		**33418·35**
	(2) 37534·58	**1864·35**	(δ_1) (2) 39404·36	**828·44**	(1) 40227·37
$m = 4$	(5) 26365·19	**1864·61**	*u*.(1) 32362·98†	**829·67**	(29059·47)
	30724·58		**32589·96**		**33420·07**
	(35083·97)	**1866·16**	(36950·13)	**831·54**	(37781·67)
$m = 5$	(5) 27696·15	**1865·51**	(1) 29561·66	**828·72**	(1) 26257·20.*u*
	30725·48		**32589·38**		**33419·32**
	(1) 33754·81	**1862·29**	(<1) 35617·10	**831·17**	($-\delta_1$) (1*n*) 36442·62

* *More probably* $\mathbf{F}_1$ (16).

† *Or* F_2 (21).

LIST of F and **F** Lines (continued).

$m = 6$	$(2\delta_1)$ (1) 28508·55	**1864·13**	$(-\delta_1)$ (<1) 30357·31	**829·19**	$(-2\delta_1)$ (<1) 31180·63
	30725·29		**32589·86**		**33417·27**
	(1) 32951·97	**1863·81** **1865·76**	(3) 34815·79 (5) 34817·73	**826·83**	(1) 35642·62
$m = 7$	(1) 29019·32	**1864·99**	(5) 30884·31	**832·82**	(5) 31717·13
	30724·37		**32588·57**		**33418·13**
	(2) 32429·42	**1863·42**	(1) 34292·84	**826·30**	(2*n*) 35119·14
$m = 8$	[29377·00]	[**1865·39**]	(1) 31242·39	**829·13**	$(-\delta)$ (1) 32048·92
	30725·60		**32590·19**		**33418·91**
	(1) 32074·21	**1863·78**	(2) 33937·99	**828·31**	(δ) (2) 34788·90
$m = 9$	(1) 29629·10	**1865·33**	(31494·43)	**831·94**	(2) 32326·37
	30724·64		**32589·98**		**33418·72**
	(2) 31820·18*	1865·36	$(-2\delta_1)$ (3) 33674·68	**825·54**	(4) 34511·08
$m = 10$	$(-\delta)$ (1) 29802·26	**1863·53**	$(-\delta_1)$ (3) 31680·16	**828·83**	$(2\delta_1)$ (4) 32525·72
	30725·20		**32589·74**		**33418·29**
	(2) 31628·35	**1865·54**	(4) 33493·89	**830·27**	(5*n*) 34324·16
$m = 11$	(29967·00)	**1863·04**	$(-2\delta_1)$ (2) 31820·04*	**832·46**	$(-2\delta_1)$ (3) 32652·20
	30725·18		**32590·25**		**33420·21**
	(5) 31483·37	**1866·08**	(2) 33349·45	**827·08**	(<1) 34176·93
$m = 12$	$(2\delta_1)$ (6) 30091·12	**1864·59**	$(-2\delta_1)$ (2*n*) 31934·91	**830·81**	$(-2\delta_1)$ (1) 32787·85
	30725·25		**32589·48**		**33419·31**
	(1) 31369·31	**1863·88**	$(-2\delta_1)$ (3) 33222·37†	**828 85**	(1) 34062·04
$m = 13$	$(-2\delta_1)$ (1) 30157·45	**1862·96**	(1) 32030·34	**831·00**	(3) 32861·34‡
	30725·48		**32589·18**		**33419·61**
	$(-2\delta_1)$ (3) 31273·55	**1864·64**	(1) 33148·03§	**829·86**	(4) 33977·89
$m = 14$	(4) 30239·16	**1862·57**	(δ_1) (<1) 32107·16		
	30724·58		**32588·06**		
	$(2\delta_1)$ 31219·95	**1864·39**	$(-3\delta_1)$ (5) 33059·49		
$m = 15$	[30297·01]	[**1865·42**]	$(-\delta_1)$ (2*n*) 32166·86	**828·06**	$(-\delta_1)$ (2) 32994·92
	[**30725·13**]				**33418·33**
	$(-\delta_1)$ (4) 31148·28		**1864 + 829·46**		$(-\delta_1)$ (1) 33841·74
$m = 16$	(1) 30348·83	**1864·97**	(<1) 32213·80		
	30725·49		**32589·56**		
	(4) 31102·16	**1863·17**	(<1) 32265·33		(δ_1) (5) 33799·54
$m = 17$	$(-\delta_1)$ (<1) 30382·49	**1861·03**	(3) 32248·49 ?		
	30724·89				
	$(-3\delta_1)$ (<1) 31047·46		**1864 + 828·48**		(1) 33754·81
$m = 18$	(1) 30421·95	**1863·31**	$(-\delta_1)$ (1) 32279·83	**829·68**	$(-2\delta_1)$ (<1) 33103·59 and $(2\delta_1)$ (<1) 33126·29
	30725·06		**32588·13**		**33418·27**
	(δ_1) (1) 31033·15	**1862·42**	(δ) (1) 32912·71		$(-\delta_1)$ (3) 33715·67 and (δ_1) (3) 33724·54

* *See* F_2 (11), *is more probably* $(-2\delta_1)$ (1) 31810·86.

† *This line is numerically* $(2\delta_1)$ $\mathbf{F}_2$ (12) *and* $(-2\delta_1)$ F_3 (22).

‡ F_3 (13) *and* $\mathbf{F}_2$ (19) *have same value.*

§ $\mathbf{F}_2$ (13) *and* F_3 (19) *have same value.*

LIST of F and **F** Lines (continued).

$m = 19$	$(-2\delta_1)$ (<1) 30444·37	**1861·79**	(1*n*) 32316·13	**831·90**	(1) 33148·03†
	30725·42		**32588·93**		**33420·61**
	(2) 30996·51	**1864·83**	(3) 32861·34*	**831·84**	(3) 33693·18
$m = 20$	$(-2\delta_1)$ (1) 30466·53	**1864·01**	(5) 32340·48	**830·69**	(δ_1) (2) 33165·52
	30525·60		**32589·36**		**33418·76**
	$(-5\delta_1)$ (1) 30949·88‡	**1863·52**	(2) 32838·25	**828·73**	(1) 33666·98
$m = 21$	(δ_1) (3) 30503·24	**1864·71**	(1) 32362·98	**828·68**	$(-\delta_1)$ (<1) 33186·10 and
					(δ_1) (<1) 33197·22
	30724·07		**32589·61**		**33418·10**
	(1) 30949·88	**1866·37**	(3) 32816·25	**828·29**	(2) 33644·54
$m = 22$	$(-\delta_1)$ (3) 30511·25	**1865·32**	$(3\delta_1)$ (<1) 32397·90§	**829·46**	$(2\delta_1)$ (3) 33222·37‖
	30725·36		**32588·60**		**33421·74**
	(5) 30933·51	**1862·19**	(1) 32795·70	**830·97**	(δ_1) (1) 33632·32
$m = 23$	[30534·18]	**1863·72**	(<1) 32397·90§		
	30725·57 + $\frac{1}{2}\xi$		**32590·16**		
	(<1) 30916·97	**1685·45**	(δ_1) (1) 32787·85		
$m = 24$	$(+2\delta_1)$ (3) 30559·83				
	30725·25				
	$(-\delta_1)$ (4) 30895·10 and	**1864·66**	(1) 32765·29§		
	(δ_1) (1) 30906·17				
$m = 25$	$(-3\delta_1)$ (<1) 30576·46¶	**1864·57**	(1) 32426·15	**829·45**	(δ) (3) 33278·20††
	30725·68		**32590·24**		
	$(-\delta_1)$ (5) 30884·81**	**1864·55**	$(2\delta_1)$ (1) 32765·29§		
$m = 26$	(<1) 30576·46	**1864·16**	$(-2\delta_1)$ (2) 32429·42§§	**829·83**	(1) 33270·45
			and $(2\delta_1)$ (3) 32451·83		
	30725·37		**32589·73**		**33420·50**
	(δ_1) (1) 30879·26	**1864·55**	$(-2\delta_1)$ (4) 32727·98	**831·70**	$(2\delta_1)$ (<1) 33581·84
$m = 27$	$(-\delta_1)$ (5) 30580·76		**1863·50 + 828·94**		(3) 33278·20
	30725·87				
	$(-\delta_1)$ (6) 30861·05		**1864·5 + 829·46**		$(-\delta_1)$ (6) 33553·33
$m = 28$	(1) 30595·63	**1864·11**	(4) 32459·74	**830·43**	$(2\delta_1)$ (3) 33301·47
			32589·87		
	[ ]		(2) 32719·41	**828·43**	$(-3\delta_1)$ (2) 33530·95‡‡
$m = 29$	(1) 30607·90		**1865·65 + 837·92**		(3) 33301·47
	30725·67		**32590·00**		**33419·00**
	(1) 30843·45	**1863·01**	(3) 32706·46	**830·14**	(2) 33530·95‡‡
$m = 30$	$(2\delta_1)$ (4) 30621·49		**1864 + 829·36**		(3) 33304·91
	[30839·22]	[**1861·57**]	(<1) 32700·79	**830·16**	(2) 33530·95‡‡

* F_3 (13) *and* $\mathbf{F}_2$ (19) *have same value.*

† $\mathbf{F}_2$ (13) *and* F_3 (19) *have same value.*

‡ *The line is* $\mathbf{F}_1$ (21) ; $\mathbf{F}_1$ (20) *may be* (4) 30976·73.

§ *Coincidences.*

‖ *This line is numerically* $(2\delta_1)$ $\mathbf{F}_2$ (12) *and* $(-2\delta_1)$ F_3 (22).

¶ *This line is also* F_1 (26).

** *This line is* F_2 (7).

†† *This line is* F_3 (27).

‡‡ *Too close to settle.*

§§ ***This line is also*** $\mathbf{F}_2$ **(7).**

The lines of the series seem to be exceptionally numerous. The results of the examination up to $m = 30$ are given in the table and the notes thereto. There are certain lacunæ—especially for $m = 4$. In these cases however corresponding displaced sets are in general observed, and naturally with large values of m this effect is more frequent. In certain cases where a set is absent a parallel set is observed linked to the normal type. This is the case for instance in $m = 4$.

The question naturally arises whether lines exist for $m = 1$. If so the formula gives a triplet with the first line at $n = 3142$, far in the ultra-red. In other spectra these values extrapolated for $m = 1$ differ considerably, often by several hundreds, from the correct ones. We can only conclude that if there are sets based on $m = 1$ they must be such that $F_1(1)$ must be in the neighbourhood of 3100. The matter can only be settled therefore by other considerations which must depend—with our present knowledge at least—either on sounding or on the presence of combination lines in the observed region. The evidence for such a triplet is given below in the notes to the list of lines. The value of $F_1(1)$ found is 3010·35 corresponding to a wave-length *in vacuo* of 33218·7 A.U. The mantissa of $3010+dn$ with the limit $30725·26+\xi$ is $989285+35·9\,(dn-\xi) = 90\,\{10998·8-·4\xi+·4dn\}-611 = 90\Delta_2-\delta$. The uncertainty in Δ_2 as found from ν_2 is too large to settle the exact value of this with so large a multiple as 90, but the fact as it stands that the mantissa differs from a multiple of Δ_2 by only a few ouns is what is to be expected if the series belongs to the F type, and so far certainly supports the more direct evidence given below for the existence of the set depending on $m = 1$. With the value of Δ_2 found below $dn = -1·5$.

A glance at the list will show that the separations observed in the second and third orders of F are less than the normal values. This points to a satellite effect. The values of ν_1 are 1862·96, 1863·92 which show deficits of 1·54, ·58 from the true value as indicated by the occurrency curve. Now a displacement by one oun produces a change of 1·25 in $m = 2$, and ·50 in $m = 3$. The deviation is then completely explained by the supposition of the existence of the satellite effect depending on δ_1. The ν_2 show similar deficits, which may possibly be due to observation errors. We should expect to find a similar effect (not necessarily the same multiple) in the order $m = 1$. In this order the oun produces a change of 4·25.

For sounders and for link evidence the data have been restricted to e, u, v links only. If we may judge from the examples of Ag and Au, the F and D linkages show a preponderance of the a, b, c, d links, and no doubt fuller evidence might have been adduced by using them, but it was necessary to set limits to the work, as well as to this communication which is long as it stands. But as examples we may give some d links belonging to the orders 2, 3 of F_1. The value of d is 1973·94. For $m = 2$ the lines (1) 20581·64, **1864·85** (3) 22446·49, are 1973·85, above F_1 and $F_1+\nu_1$, or the real F_2. The two lines (5) 23268·21, (2) 82·67 are respectively $\mp 6\delta_1$ displacements of 23275·44 which is 1875·23 (or c) above the observed F_3 or 1873·15 above the normal value $F_1+\nu_1+\nu_2$. For $m = 3$ we find

			(2) 18466·47				
			7313·17				
(1) 28341·62 **4425·90**							
	F_1	**1863·92**	F_2	**829·69**	F_3	**4426·43**	(2) 31035·76
(4) 28047·34 **4131·62**							
	1975·86		**1974·70**		**1970·10**		
	(4) 25891·58	**1862·76**	(2) 27754·34	**825·09**	(4) 28579·43	**1864·94** →	

Here F_1, 25891, 27754 form a series inequality with $d+2$, ν_1-2; F_1 forms a parallel inequality with 28341 and 28047 with $u-2$, $v-2$; 18466, F_2, 27754 a series inequality with d, e; and F_3 a parallel inequality with $v-2$, $d-2$.

As a rule in the list only displaced lines are dealt with and only occasionally linked ones. The evidence is strong for the existence of possible series with limits 30725 $(\pm x\delta_1)$—especially $x = 2$—and that for each order most of the energy goes to a line with one of these limits, not necessarily the same for different orders nor for the same triplet set, nor for the same F or **F** type.

Notes.—A displacement δ_1 produces 4·97 on $F_1(\infty)$, 5·43 on $F_2(\infty)$, and 5·65 on $F_3(\infty)$.

$m = 1$. The shortest sounder to reach from the calculated F (1) to the observed region is $2e$, giving values in the neighbourhood of 17700. In this neighbourhood we find a triplet (No. 2 of the sets on p. 383)

(6) 17638·55 **1864·64** (3) 19503·29 **829·93** (1) 20333·22,

which in separation and order of intensity corresponds precisely to the values required. With the normal value of e they indicate lines with wave-numbers at 3010·35, 4875·09, 5705·02. In connection with the question of satellite effect it may be worth noting that lines in the neighbourhood of the above may be arranged with them thus

(6) 17638	**1864·64**	(3) 19503	**829·93**	(1) 20333
		12·52		
....		(3) 19515·81		
25·53				
(5) 17664·07				

To complete the set there should be a line at 17651·31, but it was not observed by BALY. A sequence displacement in $f(1)$ of $3\delta_1$ produces a change of 12·76 and of $6\delta_1$ of 25·53. These are practically exact and correspond to the diffuse satellite arrangement with this difference, that the triplet appears as a main set in which the first line is strongest. They depend on $3\delta_1$, $6\delta_1$ displacements, so that the mantissa of the doublet set, that of 19515, is exceedingly close to $90\Delta_2$, viz., $90 \times 10997·17$.

With $F_1(1) = 3010$ should go $\mathbf{F}_1(1) = 58440·17$. This is outside the observed region on the other side, in the ultra-violet, and requires even larger sounders than F (1). The treble link $3e = 21942·30$ requires a line at 36497·87. There is no line here but (1) 36493·82 is less by 5, *i.e.* = (δ_1) **F** (1). We can arrange lines to go with it as a satellite set, viz.,

[36498·90]	**1864·50**	[38363·40]	**830**	(1) 39193·40
		17·64		**17·66**
....		(2) 38345·76	**829·98**	(2) 39175·74
25·45		**26·31**		
(3) 36473·45	**1863·64**	(1) 38337·09		

Here we find the same $6\delta_1$ for $\mathbf{F}_1$ and $\mathbf{F}_2$, whilst in F we had it only for F_1. The set with separation 17 show no $\mathbf{F}_1$, and again is numerically an exact oun multiple displacement, viz., δ which gives 17·0. The line found close to the expected—$(\delta_1)\,\mathbf{F}_1$ above—forms part of a chain

(1) 32765·29 **1864·47** (3) 34629·76 **1864·06** (1) 36493·82

It may be the representative corresponding to the satellite displacement δ_1 observed in F for $m = 2, 3$.

Using links $2e+u = 18761·38$ and $2e+v = 19056·2$ we should find lines at 39678·79 and 39383·97 with the corresponding lines to $\mathbf{F}_2$, $\mathbf{F}_3$ beyond the observed region. As a fact, we find lines at $(3n)$ 39683·95 and (1) 39386·98. The first corresponding to a $-\delta_1$ displacement should give $2e.v.\mathbf{F}_1(1) = 39679·70$. The second would differ from the same displacement by 3 and is therefore inadmissible. It should be noticed that both the $3e$ and $2e+u$ sounders give lines larger by unity than the expected value. This may, of course, be due to combined errors in the sounders and observation errors, but the **F** are entered in the list as if the sounders are correct. As none of the F (1) or **F** (1) lines can be observed, their means can have no weight for an accurate determination of the limit.

$m = 2$. The **F** lines are in the ultra-violet, and should be, with the limit chosen, at 42842·73, 44707·23. With sounder $e+v = 11742·10$ we find the set (4) 31102·16 **1863·17** (<1) 32965·33 with a small separation corresponding to that for F_1. It gives $\mathbf{F}_1 = 42844·26$ and $\mathbf{F}_2 = 44707·43$. But the set appears to be really $\mathbf{F}_1$ (16), $\mathbf{F}_2$ (16).

$m = 3$. A displacement δ_1 in $F_2(\infty)$ produces 5·42. The $\mathbf{F}_2$ line is not observed but (2) 39404 is practically an exact $(-\delta_1)\,\mathbf{F}_2$. In further illustration of this we find (2) 37431·03 linked to it by $-d$ and 5·61 (*i.e.* another δ_1) ahead in the remainder of the triplet (1) 37436·64 **828·13** $(1n)$ 38264·77. The lines in question are $(-\delta_1)\,\mathbf{F}_2$; $d.(-2\delta_1)\,\mathbf{F}_2$; $d.(-2\delta_1)\,\mathbf{F}_3$.

$m = 4$. F_2 is (1) $32362·98 - u = 28229·80$ which, as referred to above, is also (1) $26257·20 + 1972·60$ close to d link. For F_3, $(-\delta_1)$ (4) $29053·89 = 29059·54$ or $(2\delta_1)$ $(1n)$ $29071·03 = 29059·73$ or $(3\delta_1)$ (1) $29076·10 = 29059·15$. Take the mean 29059·47 as F_3. Several other lines in this neighbourhood show indications of displacement by multiples of δ_1, *i.e.* that the energy proper to F_3 has gone into a number of collaterals. None of the direct **F** are observed but there appears a parallel set displaced δ, viz., (1) 35064·09 **1864·34** (1) 36928·45 with (1) 37742·12 due to a further $3\delta_1$ displacement. The **F** lines are entered as due to these.

Both separations are about 1·6 too large and further $\mathbf{F}_1$ is about the same amount too small. Now $2\delta_1$ on the sequence term produces a change of 1·40. This would indicate that with the δ displacement on the limit concomitant displacements of $2\delta_1$ occur in the sequence so that the observed lines are $(\delta)\,\mathbf{F}_1\,(-2\delta_1)$, $(\delta)\,\mathbf{F}_2$, $(7\delta_1)\,\mathbf{F}_3\,(2\delta_1)$. The whole set again is remarkable for connection with parallel sets separated by the normal $\nu_1 \stackrel{\circ}{=} 1778$.

$m = 5$. There are observed lines for $F_{1\cdot 2}$, F_3 is entered as depending on a u link. F_2 is too large by about one unit, but $F_3 - F_1$ is normal. This order affords good evidence for the existence of the displaced sets. Consider the following system of lines:—

(δ) F	(1) 27676·52	**1863·57**	(3) 29540·09	**830·59**	(1n) 30370·68
			1·84		
		1865·41	(2) 29541·93	**828·75**	
	19·63				**11·81**
$(2\delta_1)$ F					(<1) 30382·49
F	(5) 27696·15	**1865·51**	(1) 29561·66		
	15·51				**22·26**
$(-3\delta_1)$ F	(1) 27711·66	**1863·73**	(3) 29575·39	**829·36**	(5) 30404·75
$(-\delta)$ F	(5) 27714·80	**1869·68**	(2) 29584·48		

Here 1·84 is an exact 3δ displacement in the sequent. The mesh shows series inequalities with the $\nu_1 + \nu_2 =$ normal values. The two lines in question are clearly $\pm 6\delta_1$ displacements on a normal line $(\delta)\,F_2$. On $F_1(\infty)$, δ gives 19·88, and $3\delta_1$ 14·91; on F_3, $2\delta_1$ gives 11·30 and δ 22·60. These show how closely all the conditions of the allocations are satisfied. Further, it shows how 29561 has its excess value and that the normal sequent should be the same as in $(\delta)\,F_2$, *i.e.*, 9 less, thus making its separation with F = 1864·61. The same sequent change is shown by $\mathbf{F}_2$. Both give it as $(6\delta_1)f$.

$m = 6$. Again with an even order the F lines do not appear, but there are apparent also a congery of displaced lines analogous to that in F (5). The lines given in the list give wave-numbers 28498·61, 30362·74, 31191·93. On the contrary, **F** lines are observed although $\mathbf{F}_3$ has probably been displaced. These also show evidence of displaced sets, *e.g.*, (1n) 32972·50 **1864·28** (<1) 34836·78 is 20·53 ahead of $\mathbf{F}_1$, and δ on the limit gives 19·88.

$m = 7$. The values of $F_1(\infty)$, $F_2(\infty)$ as deduced from the means are clearly too small. $F_1(7)$ is very close to the calculated value, so that if any error has been made it is probably due to the **F** which should be about 1·8 larger, and suggests a close doublet, *i.e.* a small sequence displacement as in the preceding sets. As supporting this there is a line (1) 32426·15 which as $(\delta_1)\,F_1$ would give 32431·12 making with F_1 the limit 30725·22. This corresponds clearly to the normal value. A similar displacement is also found in F_1 in the line (4) 29024·43, which as $(-\delta_1)\,F_1$ gives 29019·46 for F_1. It should be noted that the energy of F_1 has passed chiefly to the displaced line, whilst in F_2 most of its energy remains with it and a fraction passes to the displaced line. This probably means that only a small number of the normal F_2 configurations are broken up, whilst most of the F_1 are. F_3 as (5) 31717·13 gives ν_2 too large. This line and (4) 31705·47 are separated by 11·66 or a $2\delta_1$ displacement, so that there is a concomitant sequence displacement. A similar effect is shown in $\mathbf{F}_3$ with two lines (2n) 35119·14, (2) 35126·05. The lines entered appear correct for they give the normal limit, but their half difference shows a displacement in the $f(7)$ sequent. The normal line would appear to be given by $(-2\delta_1)\,\mathbf{F}_3 = 35136{\cdot}05$ making $\mathbf{F}_3 = 35124{\cdot}78$ with $\nu_1 = 831{\cdot}94$.

$m = 8$. No line is found for the calculated 29377·00, or 77·23 if we allow the same O – C as for $m = 7$. The lines (3) 29368·41 as $(2\delta_1)\,F_1$ and (1) 29403·29 as $(-5\delta_1)$ F give respectively 29378·34 and 78·45, which are larger than should be expected. The calculated value has been taken as correct. Also the lines (1) 32048·92 as $(5\delta_1)\,\mathbf{F}_1$ and (4) 32098·20 as $(-5\delta_1)\,\mathbf{F}_1$ give respectively 32073·76 and 73·36 or a mean of 73·56. But 32048 is also $(\delta)\,F_3$. This is not a mere list coincidence. As a fact $(\delta)\,F_3$ and $(5\delta_1)\,\mathbf{F}_1$ are very nearly equal, and if both existed would show as a double line too close to have been resolved. The second line has a separation 1864·23 to (2) 33962·43, and its deduced $\mathbf{F}_1$ makes with the calculated F_1 the limit 30725·18 very close to the definitive value found below. For $\mathbf{F}_2$ 33937·99 is supported by $(-2\delta_1)\,\mathbf{F}_2$ = (1) 33948·71 giving $\mathbf{F}_2$ = 33937·85, but the $F_2(\infty)$ is large and 31242 shows a separation with F_1 of 1865·39 also large. Also (1) 31247·07 as $(-\delta_1)\,F_2$ gives F_2 = 31241·64 which makes $F_2(\infty)$ = 32589·82 and the separation from F_1 = 1864·64 both improved. On the other hand $(-2\delta_1)\,F_2$ = (4) 31253·32 gives F_2 = 31242·46 precisely the line observed. These small differences depend partly on observation errors and sequence or satellite displacements. In the case of F_3 and $\mathbf{F}_3$ the equally and oppositely displaced lines give the same mean as the lines calculated from them.

$m = 9$. There seem considerable displacements in the sequences here. The calculated values for the first lines are 29632·80 and 31817·72. They are not observed, but the corresponding F_3, $\mathbf{F}_3$ lines are. There are two near observed lines (1) 29629·10 and (2) 31820·18 which give the mean 30724·64, which is small, but (1) 31810·86 = $(2\delta_1)\,\mathbf{F}_1$ would give 31820·79 and the limit 30724·94, close to the normal value. 31820·18 is then $(2\delta_1)\,F_2(11)$. With (5) 31483·37 as $(2\delta_1)\,F_2$ and ($1n$) 31505·49 as $(-2\delta_1)\,F_2$ we get respectively F_2 = 31494·23 and 94·63. The mean is entered, and a similar $-2\delta_1$ displacement gives $\mathbf{F}_2$ as entered. The normal third lines are observed. Probably the $\mathbf{F}_3$ having the same sequent as the $\mathbf{F}_2$ adopted should be that given by $(-2\delta_1)\,\mathbf{F}_3$ = (4) 34527·64 or $\mathbf{F}_3$ = 34516·37 with ν_2 = 830·83.

$m = 10$. The allocation seems satisfactory. The limits also are very close to the correct, but the different triplet separations show that the successive sequents suffer displacement, but the same in each F, **F**.

$m = 11$. The calculated F_1 is 29966·21. With (1) 29982·13 = $(-\delta_1)\,F_1$, F_1 = 29967·22. Moreover the last has links e = 7314·34 to (3) 22652·88 and u = 4133·20 to ($1n$) 25854·02 in very striking agreement. The value as calculated with normal e is entered. With the lines as entered it is seen that the means of the corresponding separations for the two series are both normal, although the individuals are abnormal. This shows that both corresponding lines have the same limit, and the same sequent, but that the latter shows a displacement from the normal value for the F_2 set. This is supported also by the fact that there are a number of close lines to $\mathbf{F}_2$. For instance, ($2n$) 33332·22 and (2) 33330·00 as $(3\delta_1)\,\mathbf{F}_2$ give respectively 33348·51 and 6·29 for $\mathbf{F}_2$. They are probably all $\mathbf{F}_2$ lines showing sequence displacements. The first gives the triplet separations 1865·14, 828·42 and limit 32589·77, the second 1862·92, 830·64 and limit 32588·66. In other words, the first gives $\mathbf{F}_2$, with same sequent as in $\mathbf{F}_1$ and F_1, the second as in F_2.

$m = 12$. Note the good agreement—the same $(-2\delta_1)$ displaced limit for F_2. $\mathbf{F}_2$ and same $2\delta_1$ for F_1 and F_3.

$m = 13$. The two displaced sets give F_1 = 30167·38, $\mathbf{F}_1$ = 31283·59. The calculated F_1 = 30166·40.

$m = 14$ *to* 30. It is remarkable how the series seems to persist to high order. It may be said that this is only apparently so, because in this region the spectrum is so crowded with lines that it is necessarily possible to select sets near the calculated values. But in truth the reason of the crowding is because of the series. The F and **F** lines crowd up together on either side of the three limits, and at the same time there are different sets of limits depending on the $2\delta_1$ displacements. The spectrum has not been examined beyond $m = 30$, and from 14 to 30 the list indicates an allocation without further specification. There is, however, much evidence not adduced here to indicate actual cases where sequence displacement occurs. The calculated values for F_1 from $m = 14$ to 30 are 30238·21, 30297·01, 30345·73, 30386·58, 30421·17, 30450·68, 30476·20, 30498·14, 30517·35, 30534·18, 30549·11, 30562·32, 30574·08, 30584·61, 30594·07, 30602·60, 30610·32. The deviation from the calculated values for $F_1(29)$ and $\mathbf{F}_1(29)$, which, however, gives the correct limit, shows that the sequent $f(29)$ receives a large displacement value, so large indeed as to totally alter its mantissa. The set must be doubtful. The whole set are more likely to be $(\delta_1)F(30)$ and $(-\delta_1)F(30)$. In fact, in this neighbourhood, the difference of two successive orders in a series is comparable with the change produced by a δ_1 displacement in the limit, and so introduces some uncertainty in allocation. It will be noticed also that, in a few cases, the same line is adduced to fit two cases, which can only happen if a line happens to be a close doublet, an unlikely supposition to happen often.

We are now in a position to determine the limits with considerable accuracy. Taking the average means where they are deduced from actual or displaced actual values, we find the limits come to 30725·340, 32589·443, 33419·079. In the first attack on the problem values of unobserved lines were deduced from observed linked lines. The corresponding mean values for the limits then found had for the last digits 5·292, 9·161, 8·918, very close to those determined from the displacements. The individual deviations from the mean are quite small for $F_1(\infty)$, considerably smaller than for the others. It is, therefore, the more reliable. The mean deviation in magnitude is ·28 and the maximum is $-$·97 in $m=7$. We may take it therefore that the true value of the F_1 limit is 30725·30 within a few decimals. The separations given by the deduced limits above are 1864·10 and 829·64.

These very accurate values afford a means of testing as to their source. If the limit were known to be a single number there could be no doubt as to its belonging to the d sequences, or as to the series being of the F type. But there is just the possibility that it may be a composite number, comprising one or more links—say, p or s terms—and that the separations may be due to oun displacements in one of them. The suspicion that this may be the case is aroused by the fact that the triplet 17615, 18447, 20312, which would be the origin of the d or $F(\infty)$ term, and in which therefore the first two lines should behave as satellites do not show complete sets with the separation 1778, 815, as they should do if normal satellites. Moreover, the intensity order with the middle line much more intense than the other is not normal. There is no test for the composite nature of 30725, but if it be really so, the most probable source would be $p=S(\infty)$, or some near collateral of this. We will therefore test this as $51025·26+\xi$, where ξ may be considerable, so as to include near collaterals, and also test 30725 as a d sequent. We will take the latter first.

At the start it may be noted that it is an argument in favour of 30725 being directly the source, that displacements by small multiples of the oun have fitted in so remarkably closely and frequently in the formation of the list of lines above.

Taking then the limits as $30725·30+\xi$, $32589·40+\xi+d\nu_1$, $33419·04+\xi+d\nu_1+d\nu_2$ the denominators are found to be

$$1·889322-30·74\xi$$

$$54831-2·60\xi+28·14d\nu_1$$

$$1·834491-28·14\,(\xi+d\nu_1)$$

$$22914-1·14\xi-1·1d\nu_1+27d\nu_2$$

$$1·811577-27\,(\xi+d\nu_1)-27d\nu_2$$

In these ξ cannot be greater than a few decimals and will produce no effect on the differences of the mantissæ, which may themselves be affected with errors ± 1 due to using 7-figure logs.

The differences may be represented as follows:—

$$5\,(10996{\cdot}8+5{\cdot}6d\nu_1-{\cdot}52\xi\pm{\cdot}2)-\delta_1 = 5\Delta_2-\delta_1$$
$$2\,(10999{\cdot}0-{\cdot}5d\nu_1+13{\cdot}5d\nu_2\pm{\cdot}5)+6\delta_1 = 2\Delta_2+6\delta_1,$$

where to a first approximation we know already Δ_2 is 10998 ± 1. The sum

$$= 7\,\{10997{\cdot}4-{\cdot}53\xi\pm{\cdot}1+4\,(d\nu_1+d\nu_2)\}+5\delta_1.$$

Quite small changes in $d\nu_1$, $d\nu_2$ can therefore make the connections with Δ_2 exact, and since the multiples 5, 2, 7 are so small any small errors in the approximate value of Δ_2 can have no effect. If we use the ν_1 as found from the occurrency curve 1864·5 $d\nu_1 = {\cdot}4$, the number in the first bracket is 10999.

The agreement with both is so close to the relation indicated that it speaks strongly in support of the D origin of the limit, and the outstanding small differences may be left for the present.

If now the other supposition be tested, viz., that the separations arise from a S (∞) source $= 51025{\cdot}29+\xi$, the three denominators are found to be

$$1{\cdot}466091-14{\cdot}36\xi$$
$$26067-{\cdot}75\xi+13{\cdot}61d\nu_1$$
$$1{\cdot}440024-13{\cdot}61\,(\xi+d\nu_1)$$
$$11160-{\cdot}31\xi-{\cdot}31d\nu_1+13{\cdot}30d\nu_2$$
$$1{\cdot}428884-13{\cdot}30\,(\xi+d\nu_1)-13{\cdot}30d\nu_2,$$

and in the 20312 set the S (∞) must enter as a negative quantity since the separations are there in inverse order. In this case ξ may be considerable. The differences may be expressed as follows:—

$$2\Delta_2+6\tfrac{3}{4}\delta-53-{\cdot}75\xi+13{\cdot}61d\nu_1\,;\ \ \Delta_2+\delta_1+9-{\cdot}31\xi+13{\cdot}3d\nu_2.$$

No permissible values of ν_1, ν_2 can make these both multiples of the oun. If it is possible to do so by a proper choice of ξ the latter must satisfy $53+{\cdot}75\xi = 153m$, $-9+{\cdot}31\xi = 153n$, where m, n are integers and 153 is the value of the oun. This requires $22 = 47{\cdot}4m-115n$. A suitable solution is $m = -2$, $n = -1$, which requires $\xi = 480$, or, say, series limit $= (11\tfrac{1}{4}\delta)$ S (∞). This method of explanation looks then improbable especially when taken with the more natural one above. It may be concluded with some confidence that the series in question is of the F type depending on D series for limits as in the usual way.

In the suggested lines for $m = 1$, found by sounding with $2e$ the mantissa of $f(1)$ was found to be $90\,\{10998{\cdot}8-{\cdot}4\xi+{\cdot}4dn\}-\delta$. ξ is small and the term involving it may be omitted. The error dn in 3010 may, however, amount to a few units because the lines on which it was based were assumed to depend on the limit 30725, whereas there is the possibility that they might belong to one of the parallel series found in the F sets depending on $(x\delta_1)$ (30725). In case, however, of dn being small and the mantissa involving the term in δ, we might expect still to find lines depending on the $90\Delta_2$, as the presence of the δ suggests satellites. To get $90\Delta_2$, requires a $+\delta$ displacement which decreases $f(1)$ by 17·02. In other words lines with wave-numbers 17·02 larger for F and less for **F**. We do not find this completely, but the following sets are observed, already given in the notes to the list under $m = 1$.

$2e.F_1$	$2e.F_2$	$2e.F_3$
(6) 17638·55	(3) 19503·29	(1) 20333·22
	12·52	
…………	(3) 19515·81	
25·53		
(5) 17664·07		

$3e.\mathbf{F}_1$	$3e.\mathbf{F}_2$	$3e.\mathbf{F}_3$
[36498·90]	[38363·40]	(1) 39193·40
5·08		
(1) 36493·82		
	17·64	**17·66**
…………	(2) 38345·76	(2) 39175·74
25·45	**26·31**	
(3) 36473·45	(1) 38337.09	

In which permissible observation errors are $dn = \pm{\cdot}7$. As has been seen the 36493 corresponds to a δ_1 displacement in the limit. The others to $3\delta_1$, $6\delta_1$, and δ in the sequent $f(1)$. The lines 38345, 39175 consequently have their sequent mantissa exactly $90\Delta_2$.

Further it was found that $(3n)$ 39683·95 is $2e.v.(-\delta_1)\ \mathbf{F}_1(1)$. The next preceding line to this is (1) 39666·49 or 17·46 behind it, again showing the required δ displacement and having the $90\Delta_2$ mantissa.

If it be granted that the series is of the F type, the limit must be a d-sequent. Consequently the mantissa of $30725{\cdot}30+\xi$ must be a multiple of the oun. Its mantissa is $889322-30{\cdot}74\xi = 81\,(10998{\cdot}13-{\cdot}38\xi)-10\delta_1 = 81\Delta_2-10\delta_1$ with great exactness. Let the true value of Δ_2 be $10998{\cdot}20+x$. Then if the relation is exact $81x+30{\cdot}74\xi+5{\cdot}7 = 0$ or $\xi = -2{\cdot}63x-{\cdot}18$, $x = -{\cdot}38\xi-{\cdot}07$. Now we know that ξ must be a small fraction, certainly $<\pm{\cdot}5$. Hence x must lie between $\pm{\cdot}2$ and $\Delta_2 = 10998{\cdot}20\pm{\cdot}20$. We should, therefore, expect this value for Δ_2 except possibly where electronic changes of atomic weight came in, as has been suggested above. If then the 1864 separations depend on exact $5\Delta_2-\delta_1$ and $2\Delta_2+6\delta_1$ we get as closer approximations $5{\cdot}6d\nu_1 = 1{\cdot}40$ or $d\nu_1 = {\cdot}25$ and ${\cdot}80+13{\cdot}5d\nu_2 = 0$ or $d\nu_2 = -{\cdot}05$, in other words $\nu_1 = 1864{\cdot}35$, $\nu_2 = 829{\cdot}59$ when the limit is 30725. When this limit is displaced by $y\delta_1$ these change by ${\cdot}45y$, ${\cdot}22y$.

Further, the conditions for $f(1)$ require $\cdot 67 - x - \cdot 4\xi + \cdot 4dn = 0$, or $dn = -1\cdot 8 - \cdot 056\xi$. If the sounder $2e$ was exactly normal this dn must be due to observation errors in 17638 of $d\lambda = \cdot 5$, but the value of e is also subject to some small uncertainty. In any case the result shows that the reference line does not depend on a displaced 30725, for if so dn would be at least 4·97.

Returning now to the discussion of the D series let us consider first this second group of clearly analogous series of lines :—

1.	(1) 19942·53	**1775·45**	(10) 21717·98	
2.	(1) 19989·72	**1780·27**	(6n) 21769·99	
3.	(1) 20017·46	**1783·71**	(10) 21801·17	
	(1) 20021·66	**1779·51**		
4.	(10) 20636·30	**1767·09**	(20) 22403·39	**813·66**→(<1)
5.	(2) 20688·96	**1785·54**	(10) 22474·44	
6.	(1) 20859·23	**1784·57**	(7) 22643·80	
7.	(4) 20962·07	**1780·09**	(10) 22742·16	**813·80**→(1)

They all, with the doubtful exception of 4, 7 have the appearance of belonging to first, or doublet, satellite sets, in which the second line is always the stronger. The 1780 separations are clearly associated with the now well recognised mid-triplet abnormality. That it is not itself a normal separation is indicated by No. 3 in which the 1779·51 also occurs.

In (1) the separation 1775·45 is $\nu_1 - 2\cdot 45$. It differs from the displaced $(\delta)\,\nu_1$ by ·29 which is within error limits. In this case the limit would be $(\delta)\,\mathrm{D}(\infty)$ which is 42·48 less than $\mathrm{D}(\infty)$ and $= 50982\cdot 81$. With this limit the mantissa of 19942 comes to $879711 = 80 \times 10998\cdot 3 - \delta_1$ or $80\Delta_2 - \delta_1$ within error limits. This is the typical form for the second satellite set of a triplet D series, but modified by the δ_1 displacement, so common in this group of elements, though here it appears in an apparently first satellite set instead of the second. We note at present that taking account of the small corrections, and writing as before $\Delta_2 = 10998\cdot 2 + x$ its true value is $80\Delta_2 - \delta_1 + 8 - 30\cdot 28\xi + 30dn - 80x$. The observation error dn is $< \cdot 2$ in this region and ξ is probably < 1.

In (2) 19989 is 47·19 above (1). The change due to the displacement δ in the limit is 42·55, whilst δ_1 in the sequent gives 5·05 suggesting that the limit of (2) is the normal $\mathrm{D}(\infty)$, with sequent $80\Delta_2$. With this limit the mantissa is found to be $879853 = 80 \times 10998\cdot 16$, or with small corrections $80\Delta_2 - 3\cdot 2 - 30\cdot 29\xi + 30dn - 80x$.

In (3) we have the modified 1783·71 with the clearly real separation 1779·51 or $\nu_1 + 1\cdot 61$. Now the displacement due to -3δ on the ν is 1·60 which is practically exact. This gives a limit 31·88 larger or 51057·17, and the mantissa becomes 879855, the same as for (2) and $= 80\Delta_2 - 1\cdot 4 - 30\cdot 29\xi + 30dn - 80x$. The line 20017·46 is 4·20 behind the other. A displacement of δ_1 in the sequent term produces 5·05. Thus 20017 is very close to a line with mantissa $= 80\Delta_2 - \delta_1$, but the difference ·85, corresponding to $d\lambda = \cdot 21$, is too great to render the relation exact.

Nos. 4 to 7, although much further towards the violet should not be put aside. They all show the exceptional separations.

In (4) the separation is $1767 \cdot 09 = \nu_1 - 10 \cdot 81$. If it corresponds to a real ν_1 the limit will be (5δ) D (∞) which reduces ν_1 by $10 \cdot 7$ and makes the limit about 212 less. The mantissa would be $82\Delta_2 + 7\frac{1}{2}\delta + 71$. This difference (71) from an exact multiple of δ_1 shows this to be impossible.

The discussion of the 1864 series has definitely shown it to be of the F type, and has given the limit within very small errors. This limit is one of the d (1) sequents of the diffuse series. Its mantissa was found to be $81\Delta_2 - 2\frac{1}{2}\delta$ or $80\Delta_2 + 15\frac{1}{2}\delta$. It gives one firm starting point for the discovery of the D series. The results just obtained indicate the lines which through their dependence on multiples of Δ_2 give the origin term of the diffuse sequence. They point, as we have seen, to the existence of several displaced, or parallel, sets of diffuse series. It is possible to show definitely that these exist, even if there be some uncertainty as to the lines occupying the position of D_{11} (1). In the normal case with a single diffuse series, the D_{11} (1) is always the strongest line of the series. Also in the normal type the D limit is the same as that of S—here $51025 \cdot 29 + \xi$. When however displacement occurs, the energy of a single line is dispersed amongst several others, and a line corresponding to the normal may be weak, or even too faint to have been observed. As a matter of experience also it is found that the lines of low order ($m = 1, 2 \ldots$) are subject to these displacements in a much greater degree than those for higher orders of m. Now there are a number of lines, which by their position and absence of ν_1 separations to stronger lines have the appearance of being D_{11} (1) lines. If they are, their mantissæ must differ from multiples of Δ_2 (in the present case $80\Delta_2$) by multiples of the oun. The fact that they may do so does not of course prove that they are D_{11} lines. If they do not do so it proves that they are not. They may however in the latter case belong to a displaced series, satisfying the multiple law when the proper displaced limit $(y\delta_1)$ S (∞) is employed. This gives us a method of testing as to what displacement a given line may correspond. If our calculus were already fully established the next step would be to apply this test to the above lines. But in reality we are testing our calculus to see if it can be firmly established, and our immediate aim must be to obtain independent evidence for the existence of parallel series. For this immediate purpose it will only be necessary to apply the test to two lines, the general question being postponed for the present.

In the first attempt at arranging the D_{11} series the strong lines (8) $20559 \cdot 08$ and (10) $38366 \cdot 36$ were taken for $m = 1, 2$, and the formula calculated with the limit D (∞) = S (∞). As will be seen immediately, this gave satisfactory agreement with sounded observed lines up to $m = 15$; and as a matter of fact this series was used to test for the parallel sets displaced $(\pm 2\delta_1)$ S (∞) on either side of it. Now the formula constants for a set only vary slightly if the wave-number of the line chosen for $m = 1$ is changed considerably. This therefore did not prove definitely that 20559 is the correct D_{11} (1), and as a fact it does not satisfy the multiple test. Its mantissa is $897337 - 31 \cdot 14\xi$. That of the line 19989, which is shown above to be the origin of the normal D set is $879853 - 30 \cdot 29\xi$. The difference is $17484 = 28\frac{1}{2}\delta + 71$. This is as far as it can be from being a true multiple, and it is quite impossible to explain this by any observation errors in the two lines. For instance our maximum admitted error can only change the mantissa by 6. If the test is valid therefore 20559 cannot be $D_{11}(1)$. Yet it has all the appearance of such a line. Is it a displaced one? Suppose it corresponds to $y\delta_1$. Now a displacement of δ_1 on the limit changes its value by 10·62, so that $(y\delta_1)\,D(\infty) = 51025{\cdot}29+\xi-10{\cdot}26y$. If p denote the ratio of actual observation error to the maximum permissible, *i.e.*, $O = d\lambda = {\cdot}05p$, the mantissa of 20559 with the new limit is $897337+330{\cdot}7y+6p-31{\cdot}14\xi$. The denominator of 19989 is $80\Delta_2 = 879856+80x$. Also $330{\cdot}7 = \frac{1}{2}\delta+25{\cdot}7$. Hence the mantissa of 20559 is

$$80\Delta_2-80x+17481+2y\delta_1+25{\cdot}7y+6p-31{\cdot}14\xi$$
$$= 81\Delta_2+10\tfrac{1}{2}\delta+2y\delta_1+68-81x+25{\cdot}7y+6p-31{\cdot}14\xi$$

in which x is small (about $\pm{\cdot}2$). Also from the consideration of 19989 above

$$-3{\cdot}2-80x+6p'-30{\cdot}29\xi = 0.$$

Eliminating x

$$71+25{\cdot}7y+6\,(p-p')-{\cdot}47\xi = M\,(153)$$

in which p, $p'<1$ and ξ cannot exceed about 2.

It is not possible to satisfy this with $y = 0$, ± 1, or ± 2.

With

$$y = 3 \qquad -6+6\,(p-p')-{\cdot}47\xi = 0,$$
$$y = -3 \qquad -5+6\,(p-p')-{\cdot}47\xi = 0.$$

If, then, 20559 be a D_{11} line it belongs to one of $(\pm 3\delta_1)\,D(\infty)$, and is definitely excluded as a possible normal D_{11}.

The next line of higher frequency is the weak line (1) 20581·64. Its mantissa is $898040-31{\cdot}17\xi = 81\Delta_2+11\tfrac{3}{4}\delta+7-81x+6{\cdot}5p-31{\cdot}17\xi = 81\Delta_2-11\tfrac{3}{4}\delta$ within error limits. This therefore passes the D_{11} test. If it is the actual D_{11} its weak intensity is due to the numerous displacements for $m = 1$. If it be taken as $D_{11}(1)$ with the previously mentioned (10) 38366·36, and the limit $D(\infty)$, the series formula is found to be

$$n = 51025{\cdot}29-N\Big/\left\{m-{\cdot}988854-\frac{090814}{m}\right\}^2.$$

The lines after $m = 2$ lie in the violet outside the observed region. To test them therefore recourse must be had to sounding, only the *e.u.v.* links have been used for this purpose. The results are given in the middle column of the subjoined table and exhibited in diagram (Plate 3). Details are given in the notes following the table. Lines were calculated down to $m = 15$ and tested. The result may be regarded as conclusive in establishing the series, as well as increasing confidence in the method of sounding—a confidence which reposes not on a single coincidence, but on the recurrence of a large number of successive ones. As will be seen the agreement between the calculated values and those found by sounding is remarkably close. The only doubtful case may be that for $m = 1$. If 20559 be taken for this the formula is only slightly changed and the agreement almost as good, the calculated value for $m = 3$ giving O−C $d\lambda$ = ·05 instead of ·00. It is excluded because it cannot belong to a diffuse series in which the limit is S (∞).

Table of Parallel XD Series.

After $m = 2$ the wave-numbers at the head of each set are those calculated from the formula and are not, as usual, enclosed in [ ]. The links are entered as attached to the respective observed lines. On the right of each is entered the O−C ($d\lambda$) calculated as if the error were on the observed line. They would be less—often considerably less if calculated on the series line itself.

m.	($2\delta_1$) D (∞).		D (∞).		($-2\delta_1$) D (∞).	
1	[20550·58], = ($-2\delta_1$)	**31·06**	(1) 20581·64	**31·06**	[20610·70], = ($+2\delta_1$)	
	e.(2) 27864·16	·06			e.(6) 27926·53	−·22
	u.(6) 24683·37	·06			u.(<1) 24749·35 ($-\delta_1$)	−·11
	v.(2) 24977·40	−·18			v.(5) 25036·99	·26
2	(2) 38345·76*	**20·60**	(10) 38366·36	**21·20**	[38387·61]	
	(1) 31033·15 (δ_1).e	−·02			(<1) 34251·50.u	·02
	(5) 33915·20 ($-2\delta_1$).v	·00			(2) 33062·43.v	−·02
	or				v.(5) 35499·89.e	·12
	[38345·11]	**21·25**			(26940·63).e.u	−·05
	(<1) 34212·13.u	−·01			or [38390·24] = ($2\delta_1$)	
	(4) 33908·99 ($6\delta_1$).v	−·09			(<1) 31076·51.e	−·07
3	44004·50 = $-\delta_1$	**21·94**	44026·48	**21·76**	44048·20	
	(<1) 36690·52.e	−·01	(1) 36712·34.e	0	u.(1) 40867·04.e	·01
	(1) 32262·65.e.v	−·02	(2) 39598·95.v	−·02	(2) 32304·23 ($-2\delta_1$).e.v	·03
4	46557·99	**20·24**	46578·23	**21·43**	46599·66	
	(3) 34815·79.e.v	·00	[39264·13.e]		(1) 31971·46.2e	·00
			(<1) 34836·78.e.v	−·05	(1) 27836·93.2e.u	·17
			(35130·95).e.u	0		
5	47927·39	**21·39**	47948·78	**21·45**	47970·23	
	(<1) 28871·19.2e.v	·00	(<1) 33322·45.2e	−·18	(2) 36523·41.e.u	−·03
	(3) 33301·47.2e ?	−·20	(3) 36209·04.e.v	−·24	(3) 36229·78.e.v	−·13
			(1) 29189·92.2e.u	−·29		
6	48748·78	**21·35**	48770·13	**21·35**	48791·48	
	(1) 37300·91.e.u	·04	(<1) 34140·88.2e	·09	(4) 34163·39.2e	·09
	(3n) 29692·79.2e.v	−·02	(1) 29710·88.2e.v′	−·06	v.(3) 38589·92.2e	·09
			(2) 37320·68.e.u	·15		
			(5) 30005·34.2e.u′	−·01		
7	49280·00	**21·25**	49301·25	**21·25**	49322·50	
	(3) 37829·35.e ($-\delta_1$).u	·00	(1) 34870·17.2e	·24	u.(1) 30559·83.2e	·14
	(2) 37534·58.e ($-\delta_1$).v	·00	(3) 37849·82.e.u	·29		
	v.(1) 39082·95.2e	−·21	(<1) 37554·87.e.v	·32		
			(<1n) 30444·37.2e.v	·07		

* This is also 3e.$\mathbf{F}_2$ (1) (δ) the 90 Δ_2 linked $\mathbf{F}_2$. It is probably not ($2\delta_1$) D (2) but hides it.

TABLE of Parallel XD Series (continued).

m.	($2\delta_1$) D (∞).		D (∞).		(− $2\delta_1$) D (∞).	
8	49643·23 (3) 37901·76.*e.v* (<1) 35011·06 15·47.2*e* (4*n*) 35019·89 (1) 30879·26 81·78.2*e.u* (1) 30884·31 (2) 30588·24.2*e.v*	**21·25** − ·04 − ·03 ·00 − ·13	49664·48 (<1) 35037·31.2*e* (1) 38217·68.*e.u* (1) 30607·90.2*e.v*	**21·25** − ·08 − ·03 ·04	49685·73 *u*.(1) 39193·40.2*e* ?	 − ·19
9	49902·63 (1) 38159·20.*e.v* (3) 31139·94.2*e.u*	**21·25** ·09 ·13	49923·88 (35295·68).2*e*	**21·25** 00	49945·31 (1) 38203·51.*e.v* *v*.(1) 39745·63.2*e*	 − ·03 − ·04
10	50094·30 *u*.(2) 39598·95.2*e* (2) 31035·76.2*e.v* *or* *e.e* (δ_1).*v*	**21·25** ·02 ·24 ·0	50115·55 (1) 38666·62.*e.u* (31355·28).2*e.u*	**21·25** ·11	50136·80 (1) 38689·81.*e.u* (<1*n*) 31076·84.2*e* (− δ_1).*v*	 − ·03 ·07
11	50239·95 (3) 38791·71.*e.u* (4) 35610·00.2*e* *u*.(1) 39745·63.2*e* (3) 31479·21.2*e.u* (<1) 31180·63.*e.e* (− δ_1).*v*	**21·25** ·06 ·14 − ·04 − ·07 ·09	50261·20 (1) 35637·28.2*e*(δ_1) (1) 38521·55.*e.v*	**21·25** ·03 ·16	50282·45 (1*n*) 38835·86.*e.u* *v*.(3) 40082·56.2*e* (3) 31224·82.*e.e* (− δ_1).*v*	 − ·04 ·00 ·01
12	50353·24 *u*.(1) 38906·95.*e* (1) 31592·28.2*e.u*	**21·25** − ·06 − ·04	50374·49 (2) 35745·69.2*e* (1) 38632·72.*e.v*	**21·25** ·05 − ·02	50395·74 (5) 35767·81.2*e* (1) 31636·86.*e.e* (δ_1).*u*	 − ·02 − ·02
13	50443·04 (3) 31680·16.2*e.u.* (5) 31384·86.*e.e* (− δ_1).*v*	**21·25** ·15 − ·03	50464·29 (1) 35836·52.2*e* (1*n*) 38720·67.*e.v* *u*.(2) 39969·62.2*e*	**21·25** − ·03 ·10 ·02	50485·54 (<1) 38935·20 38·00.*e.u.* (1) 38940·81 (6) 31725·89.2*e.u* (δ_1) (<1) 31431·51.2*e.v* (δ_1)	 ·01 ·00 − ·03
14	50515·50 (3) 38970·66.*e* (− δ_1).*u* (1) 35889·37.2*e* ? (1*n*) 31755·50.2*e.u*	**21·25** ·02 − ·16 − ·13	50536·75 (3) 31479·21.2*e.v*	**21·25** ·13	50558·00 (1) 39107·56.*e* (− δ_1).*v* (1) 35929·09.2*e* *u*.(1) 40063·94.2*e*	 ·05 ·06 − ·06
15	50574·68 (1*n*) 38835·86.*e* (δ_1).*v* (1) 35949·76.*e* (δ_1).*e* *u*.(3) 40082·56.*e* (δ_1).*e* (5) 31717·13.*e* (δ_1).*e.u*	**21·25** − ·06 − ·07 − ·02 − ·05	50595·93 (2) 35963·72.2*e* *or* 2*e*(− δ_1) (1) 38852·76.*e.v*	**21·25** ·30 − ·05 ·06	50617·18 (3) 35985·33.2*e* (− δ_1) (1*n*) 31755·50.2*e.u* (<1) 31559·98.2*e.v*	 − ·07 ·03 ·10

Notes on Table of D (∞).—*m* = 3. The *e* linked line has separation 1780·29 to (4*n*) 38492.
m = 4. The *e* linked line is not observed, but it would be separated 1781·55 from (1*n*) 41045.
35130. There is no line to this, but it seems split into two as indicated in the scheme

(2) 35126·05					(1) 37720·05	
	31·05	**1776·95**	(3) 36908·00	**816·17**		24·17
(2) 35136·05					(1) 37728·30	

In which it may be noticed that the sum of the separations is 2593·12, the normal value. The order m is too large to definitely settle the sequence displacements if in them. They would be close to $\pm 18\delta_1$ for the first and $\pm 14\delta_1$ for the third. But if the connection is real a better explanation might be modification of the links, $\pm \delta_1$ on $e.u$ give the exact numerical agreement for the two D_{33} lines, and $e(\pm\delta_1)$, $u(\pm 2\delta_1)$ would give 5·86 where 6 is observed.

$m = 5$. 33322 has separations **1782·14** (4), **811·06** (4), the sum being normal.

$m = 6$. 34140 has separations 1782·92 to (2) 35923. Further, there is 30005 **1780·45** (1) 31785 **813·4** (1) 32599, in which the sum of the separations is closely normal. The $2e.u$ and $2e.v$ linked lines differ by 3·41 and 3·05 from the calculated, but are correct if the links $u' = u(-2\delta_1)$ and $v' = v(-2\delta_1)$ are taken. They are inserted since the corresponding normal links occur in other lines and the amounts are exact.

$m = 7$. The $2e$ linked 34870 has triplet separations **1784·44** (3), **813·73** (3), also the $2e.v$ has **1778·14** to intensity (<1). The $e.u$, $e.v$ linked lines are inserted although their difference as they stand are so considerable, because they all are exact if the links involved are taken as displaced $(-\delta_1)$.

$m = 9$. There is no observed $2e$ linked line, but it seems to have split up into two, thus

(<1) 25293·16		**1778·20**	(4) 37071·36
	95·65		
(<1) 25298·14			

The two observed lines are numerically $D_{11}(9)(\pm\Delta_2)$.

$m = 10$. The $2e.u$ line is split up into two (<1) 31350·91, (<1) 31359·66, of which the former shows **1778·79** (4), **811·75** (5).

It should be noticed how many of the e and $2e$ linked lines introduce the modified triplet separations.

Notes on Table of $(\pm 2\delta_1)$ D.—$m = 1$. The displacements on the limits would give 20560·40 and 20602·89. There is the already noted 20559 near the first discarded for $D(\infty)$ because it does not pass the multiple test. It serves better for $(-2\delta_1)$ D, but would require at least an observation error $d\lambda = -\cdot 1$ which we have regarded as excessive. There are no observed lines connected by e, u, v links to either, nor near them. Those given in the table are, however, very clear. They make the sequence term displaced $2\delta_1$ from that of the D series, viz., $-2\delta_1$ for $+2\delta_1$ on limit, and $+2\delta_1$ for $-2\delta_1$ on limit, *i.e.*, interchange of $\pm 2\delta_1$ for $\mp 2\delta_1$ on limit. In the third series the e and v linked lines differ respectively by $e + 1\cdot 74$ and $v - 1\cdot 70$. They form therefore a parallel inequality, and are good evidence in spite of the considerable difference 1·7.

$m = 2$. The limit separation should give for the first series 38345·11, and it apparently exists although possibly it belongs to a series commencing with 20559. There is clear evidence of a set corresponding to $m = 1$, shown by sounding and giving a sequence displacement of $-6\delta_1$. In the third series the line depending on the limit change alone would be 38387·56. This gives links $u + 2\cdot 85$ and $v - 2\cdot 87$ with the lines indicated or a parallel inequality. They are explained by $\pm 2\delta_1$ displacements in the sequent.

$m = 3$. Here δ_1 as a displacement in the sequent produces a separation of ·5. The sequent displacement in the first series is therefore $-\delta_1$ and $+\delta_1$ in the third. The line 32304 however shows $-2\delta_1$.

$m = 7$. Modified links $e(-\delta_1)$ are introduced. This is supported by the fact that the two lines given differ respectively by 2·37, 2·32 from values given by normal e, whilst the modification of e by δ_1 produces 2·32. The double example and exact difference give weight to the suggestion.

$m = 10, 11$. Similarly the modified e makes exact agreement, and they enter in a corresponding way and in both series.

We can now use this series to test the question as to the existence of parallel series depending on $(\pm 2\delta_1)$ D (∞). This does not mean that the sequences must be the same in each. In fact it is to be expected that there may be a concomitant change in them also, but they can only differ by a few multiples of the oun. The important point to notice is that for large values of m, the effects of such differences become negligible and the observed separations from the standard D should approximate to $\pm 21\cdot 25$, which is that due to $2\delta_1$ on the limit. It will not be necessary to go into such a full discussion as in the standard series with D (∞), except for the first three sets, where the evidence for changed sequence terms with displaced limits is conclusive and important. The lists and sounders are given in the same table as for the standard series, in the first and third columns respectively. The considerations adduced enable us to feel that the ground is safe in recognising that parallel series depending on displacements on the limit D (∞) = S (∞) really exist. The evidence does not depend on a single numerical coincidence of a line, or of a line found by sounding, but on the fact that these numerical coincidences appear for so many sounding links in all the 15 sets tested. This is not affected by the high probability that several of the sounded lines are chance coincidences. This result then gives more confidence to the method partially applied above, in the application of the law that the D sequences must have mantissæ which differ by multiples of the oun from multiples of Δ_2, or in other words must be themselves multiples of the oun. This method consisted in testing certain lines to see whether by using the displaced limits $(y\delta_1)$ D (∞)—now seen to really exist—the above relation holds.

The evidence seems to show that the typical lines—D (∞) = S (∞)—for $m = 1$ have been much affected by displacement effects, and that consequently the intensities of the normal lines themselves are much diminished. Although this is some disadvantage, it will be well to attempt here to get some insight into the complete satellite system for the first two orders.

We have seen that 19989 belongs to this normal set with a mantissa $= 80\Delta_2$ and that 20581 satisfies the condition necessary for a D_1 line with this. The difference of their mantissa (see below) is $29\frac{1}{4}\delta$. If they are of the D_{12}, D_{11} types, as is indicated by the fact that the first belongs to a doublet and the second stands by itself, a triplet satellite set should be expected whose first line D_{13} is separated from the D_{12} by about three-fifths that of D_{12} from D_{11}. Its mantissa should therefore be about $18\delta = \Delta_2$ less. This would mean a line about 19623 forming the first line of a triplet. No line is observed here. There are, however, lines at (1) 19602·66 and (3) 19632·44 of which 19602 passes the suitability test for a normal D line, and the other does not. The mantissa of 19602 is $79\Delta_2 - \delta$, *i.e.*, 19δ behind that of 19989 and rather too large. On the other hand the problematic 19623 may be too weak, in which case the corresponding D_2, D_3 lines which should be stronger might be observable. The D_2 line should be about 21400. We do find this, in fact, with triplets of a kind. The whole set of these lines can then be arranged as follows :—

(1) 19602·66	**1774·45**	(3n) 21377·11	**821·04**	(2) 22198·15
δ				
[19623·05]	**1777·90**	(2) 21400·95	**809·53**	(2) 22210·48
18δ				
(1) 19989·72	**1780·27**	(6n) 21769·99		
29$\frac{3}{4}$ δ				
(1) 20581·64				

in which it may be noticed that in the first the sum of the separations is the same as 1780·19+815·30, *i.e.*, the modified $\nu_1+\nu_2$, and in the second 809·53 = 815·20−5·67 whilst a δ_1 displacement on the sequent produces a change of 5·14. With the limit 51025·29+ξ the mantissæ of the d sequents are—$d\lambda$ = ·05p—

$$19602,\quad 868240-29{\cdot}73\xi+6p = 79\times10998{\cdot}2-618+6p-29{\cdot}73\xi$$
$$19623,\quad 868846-29{\cdot}75\xi+6p' = 79\times10998{\cdot}2-12+6p'-29{\cdot}75\xi$$
$$19989,\quad 879853-30{\cdot}29\xi+6p = 80\times10998{\cdot}2-3+6p-30{\cdot}29\xi$$
$$20581,\quad 898040-31{\cdot}17\xi+6{\cdot}5p = 80\times10998{\cdot}2+18184+6{\cdot}5p-31{\cdot}17\xi$$

in which p lies between ±1 and p' depends on error of extrapolated line, and may be >1. Writing as before 10998·2 = Δ_2-x these become respectively

$$79\Delta_2-\delta-7+6p-79x-29{\cdot}73\xi$$
$$79\Delta_2\quad-12+6p'-79x-29{\cdot}75\xi$$
$$80\Delta_2\quad-3+6p-80x-30{\cdot}29\xi$$
$$80\Delta_2+29\tfrac{3}{4}\delta+7+6{\cdot}5p-80x-31{\cdot}17\xi$$

The multiple rule requires that the last four terms in each expression must vanish. This is clearly possible for small values of p, say $<\frac{1}{2}$, and a single relation between x and ξ, say $8x = -3\xi$.

As a further test of the reality of the extrapolated line 19623 linked lines may be sought for. There is none for $+e$, but lines are found close for u, v, $e\pm v$, viz.,

(6) 23754·27	**1778·40**	(3) with	$u-2{\cdot}1$	19623·19
(<1n) 24053·37		,,	$v+2{\cdot}1$	19623·16
(2) 24509·92		,,	$e-v$	19623·82
(2) 31362·71		,,	$e+v$	19620·66

Taking 19623 as D_{13}, the satellite differences are $29\frac{3}{4}\delta$ and 18δ. Since $18\times5 = 90$ and $29\frac{3}{4}\times3 = 89{\cdot}25$, these separations are very closely in the ratio 5 : 3 in accordance with the rule for the known triplet series in other groups. The triplet set 19602 appears somewhat anomalous. The middle line appears to have the modification so common in the middle set of a triplet, but in the opposite direction to the usual one, *i.e.*, the sequent to the second line is $d(-\delta_1)$ instead of $d(\delta_1)$.

The preceding considerations have shown the existence of a complete set of D (1) satellites. But further, the D suitability test shows that 20305·60 may be a D_1 line with normal limit and that with it may go two sets of extrapolated triplets in which the mantissa of the first is a multiple of Δ_2. They are

[16013·45]	**1777·90**	(1) 17791·35	**816·44**	(8) 18607·79
	1784·83	(1) 17798·28	**809·51**	
[17725·39]	**1777·90**	(3) 19503·29	**816·96**	(4) 20320·25
(1) 20305·60				

The first set, however, involves for the D_{33} the line 18607, which belongs to the 1864 F set discussed above, and is stronger than we should expect. Provisionally we will suppose the true D_{33} is hidden by the F line. The line 20320 has been already adduced (p. 383) as showing connection with the 1864 separations in a similar way to 20312. The mantissæ of the D_1 lines are (p' indicating extrapolated lines), $769890 = 70 \times 10998{\cdot}2 + 16 + 3{\cdot}2p' - 25{\cdot}2\xi = 70\Delta_2 + 16 + 3{\cdot}2p' - 70x - 25{\cdot}2\xi$, $814815 + 4{\cdot}2p' - 27{\cdot}25\xi$, and $889493 + 6p - 30{\cdot}75\xi$. They differ successively by 44925 $= 4 \times 10998{\cdot}2 + 1\frac{1}{2}\delta + 16 + 4{\cdot}2\,(p'_2 - p'_1) - 2{\cdot}0\xi = 73\frac{1}{2}\delta + 16 + 4{\cdot}2\,(p'_2 - p'_1 - 4x - 2{\cdot}0\xi$ and $74679 = 122\frac{1}{4}\delta - 17 + 6p - 4{\cdot}2p'_2 - 7x - 3{\cdot}5\xi$. The conditions are satisfied within error limits that the mantissa of the extreme satellite is $70\Delta_2$, and that the differences for the satellites are due to $122\frac{1}{4}\delta$ and $73\frac{1}{2}\delta$. Since $122\frac{1}{4} \times 3 = 366{\cdot}7$ and $73\frac{1}{2} \times 5 = 367{\cdot}2$, the normal ratio of satellite separations is again reproduced. The evidence is clear, therefore, for two groups of normal diffuse series depending on $79\Delta_2$ and $70\Delta_2$ respectively.

The fact that 20320 is connected with 1864 in the same way as 20312, that the ν_2 separation is not good, and that we should expect a doublet here in place of a triplet rather point to the supposition that it is not a member of the set. For the D_{13} line the sequent is $51025{\cdot}29 - 16013{\cdot}45 = 35011{\cdot}84$, on this the oun displacement produces a change of 6·04 which accounts for the modified $\nu_1 = 1784{\cdot}83$ in the usual way. The mantissæ of the two D_{11} groups 20581, 20320 differ by 14δ.

It will be sufficient here to attempt the allocation of the corresponding satellites for $m = 2$ only. They should be at about the same oun multiple distance from the D_{11} line as for $m = 1$. The $D_{11}(2)$ has been taken as 38366 with denominator 1·943447. Taking the first group, the satellites should have denominators about $29\frac{3}{4}\delta$ and $47\frac{3}{4}\delta$ less than this. These would correspond to 38208·61 and 38111·69. With regard to the first the lines in this neighbourhood are (1) 38199·58, (1) 38203·51, (1) 38217·68, of which the first and third are respectively 9·03 less and 9·07 greater than 38208, and suggest equal displacements on either side. They are found to correspond to $31\frac{1}{2}\delta$, 28δ from D_{11}, or $\pm 7\delta_1$ on either side of 38208, with errors

$d\lambda = \pm \cdot 02$, whilst 38203 corresponds to $30\frac{3}{4}\delta$ with $d\lambda = \cdot 01$. They should form portions of doublets with lines at 39977·48, 39995·58. None have been observed, but it must be remembered that these are close to the end of the observed region where only the stronger lines would be seen. Sounding with $-(e+u)$ we find lines (1) 28530·91, (3) 28548·26 respectively, $e+u-\cdot 71$ and $e+u+\cdot 04$ behind the expected D_{22} lines showing no lines at ν_2 ahead, which should be visible if they existed. We may, therefore, conclude the two lines in question belong to a doublet set. With regard to the suspected D_{13} line (38111) we find in this neighbourhood

(1) 38134·61	**1780·23**	[39915·84]	**815·20**	(1*n*) 40731·04
(1) 38129·37	**1778·21**	(1) 39907·58	**810·36**	(1) 40717·94
(1*n*) 38108·60				

Calculation shows that the first lines of these sets give with $D_{11}(2)$ separations depending on displacements $43\frac{1}{2}\delta\,(d\lambda = 0)$, $44\frac{1}{2}\delta\,(d\lambda = \cdot 01)$, $48\frac{1}{4}\delta\,(d\lambda = -\cdot 03)$. All that can be said is that these lines may be the required satellite sets. As, however, a displacement of $\frac{1}{2}\delta_1$ only produces a change of about ·05 in λ it is not possible to get any certainty. It is further possible that some of the set may belong to parallel series. For instance $38108 = (2\delta_1)\ 38129$. The first two are, however, so close $d\lambda = \cdot 00$ and ·01 that they are entered as D_{13} lines.

For the second group 20305 is a 14δ displacement from 20581. We should expect the corresponding $m = 2$ line about the same displacement from 38366. This is satisfied by the line (1) 38292·32 giving 14δ with $d\lambda = -\cdot 01$. There is also a doublet (1) 38285·86, **1778·08**, (1) 40063·94 with normal separation and displaced $15\frac{1}{4}\delta$ with $d\lambda = \cdot 00$. The first D (1) is displaced $122\frac{1}{2}\delta$ from its D_{11} line 20305 and the second by an extra $73\frac{1}{2}\delta$. If the $D_{11}(2)$ line is taken as 38285 the calculated line with $122\frac{1}{2}\delta$ is 37611·59. There is no line here, but there is a doublet, which with this line may be written

(2) 37606·15	**1780·83**	(1) 39386·98
[37611·59]	**1775·39**	

The mean of these two separations is 1778·11 or the normal ν_1. We have here a parallel inequality due to $2\delta_1$ displacement in 39386 and an extra $2\delta_1$ in 37606, or 37606 is $123\frac{1}{2}\delta$ ahead of D_{11}.

Again calculating the $73\frac{1}{2}\delta$ displacement on 30706 the line should be 37174·40. It is not observed but there is a triplet close to it showing a similar inequality to the former. It is

(1) 37159·39	**1774·38**	(1) 38940·81	**813·67**	(<1) 39754·48
(1*n*) 37166·43	**1781·42**			

The mean of the two first separations is 1777·90 or exact ν_1 and we have an exact parallel inequality. The inequality is due to two successive $1\frac{1}{4}\delta$ displacements, and 37159 is exactly $2\frac{1}{2}\delta$ extra on the calculated 37174, or 76δ from the first satellite.

The first two orders of the two groups are represented in the following scheme in which the satellite separations are given as sequent displacements from d_{11} :—

$m = 1$.

The $79\Delta_2$ group.

[19623·05]	(2) 21400·95	(2) 22210·48
$47\frac{3}{4}\delta$		48δ
(1) 19989·72	(6*n*) 21769·99	
$29\frac{3}{4}\delta$		
(1) 20581·64		

The $70\Delta_2$ group.

	[16013·45]	(1) 17791·35	(8) 18607
	$195\frac{3}{4}\delta$		
	[17725·39]	(3) 19503·29	
	$122\frac{1}{4}\delta$		
14δ	(1) 20305·60		

$m = 2$.

The $79\Delta_2$ group.

(1) 38129·37	(1) 39907·58	(1) 40717·94
$44\frac{1}{2}\delta$	$44\frac{1}{2}\delta$	$46\frac{1}{4}\delta$
(1) 38199·58		
$31\frac{1}{2}\delta$		
(1) 38217·68	(3) 28548·26.*e.u*	
28δ	28δ	
(10) 38366·36		

The $70\Delta_2$ group.

	(1) 37159·39 }		
	$199\frac{1}{2}\delta$ }	(1) 38940·81	(<1) 39754·48
	(1*n*) 37166·43 }		
	197δ	$198\frac{1}{4}\delta$	$198\frac{3}{4}\delta$
	(2) 37606·15	(1) 39386·98	
	$123\frac{1}{2}\delta$	$122\frac{1}{2}\delta$	
14δ	(1) 38285·86		

Without dealing with the whole of the material at disposal we will illustrate its application by considering in more detail the portion of the spectrum given on p. 382 in which the majority of the lines undoubtedly belong to D (1) systems. It is to be noticed that the effectiveness of the method in the present case depends on the facts, (1) that the observation errors do not exceed $d\lambda = \cdot 05$, and (2) that with $m = 1$ it is consequently possible to determine the values of the mantissæ to within 6 units in the sixth significant figures, whilst a displacement of one oun in the sequent produces a change in λ of the order 1·2, or twenty-four times the maximum observation error. The limit 51025 being supposed displaced by $y\delta_1$ becomes $51025{\cdot}29 - 10{\cdot}62y + \xi$. The mantissæ of the sequences are then calculated with this limit, and expressed in terms of Δ_2, δ_1, x and p where $\Delta_2 = 10998{\cdot}2 + x$, and $-p$ is the ratio of the observation error to the maximum ($d\lambda = \cdot 05$). The series more fully discussed above is definitely taken as depending on the limit $y = 0$. In other words the mantissa of 19989 is exactly $80\Delta_2$ which condition requires, writing q for its p,

$$3{\cdot}2 + 30{\cdot}29\xi - 6q + 80x = 0,$$

and gives a relation between ξ and x. The term in x in each mantissa is then eliminated by means of it. There can be little doubt about the allocation of 19889, but even should it be in error the doubt does not affect the argument as to the relative displacements of the different lines, as $y\delta_1$ denotes the displacements relative to 19889. The final results are given in the following table :—

1	19880	$80\Delta_2+2y\delta_1-5\frac{1}{2}\delta+66+14y+\cdot 2\xi+6(q-p)$	**–4,**	11	**–5,**	–4
2	19942	$80\Delta_2-\delta_1$ with $y=4$	**4,**	11		
3	19959	$80\Delta_2+2y\delta_1-\delta-61+16y+\cdot 05\xi+$,,	**4,**	3		
4	19989	$80\Delta_2$ with $y=0$	**0**			
5	20017	$80\Delta_2+2y\delta_1+1\frac{1}{4}\delta+77+17y-\cdot 05\xi+$,,	**–4,**	9	**4,**	–8
6	20021	,, $+1\frac{1}{2}\delta+52+17y-\cdot 05\xi+$,,	**–3,**	1	**6**	
7	20029	,, $+2\delta-31+17y-\cdot 05\xi+$,,	**2,**	3		
8	20041	,, $+2\frac{1}{2}\delta+50+17y-\cdot 07\xi+$,,	**–3,**	–1	**6,**	0
9	20080	,, $+6\frac{1}{2}\delta-2+18y-\cdot 11\xi+$,,	**0,**	–2		
10	20107	,, $+10\delta-33+18y-\cdot 17\xi+$,,	**2,**	3		
11	20305	,, $+15\frac{3}{4}\delta+14+21y-\cdot 46\xi+$,,	**0,**	14	**–1,**	–7
12	20312	,, $+16\delta+82+21y-\cdot 47\xi+$,,	**–4,**	–2	**3,**	–8
13	20320	,, $+16\frac{1}{2}\delta+9+21y-\cdot 47\xi+$,,	**0,**	9		
14	20333	,, $+17\frac{1}{4}\delta-50+22y-\cdot 50\xi+$,,	**2,**	6	**–5,**	–7
15	20443	$81\Delta_2+2y\delta_1+4\frac{3}{4}\delta-11+24y-\cdot 30\xi+$,,	**0,**	–11	**–6,**	–2
16	20454	,, $+5\frac{1}{4}\delta+41+24y-\cdot 30\xi+$,,	**–2,**	–7	**5,**	8
17	20467	,, $+6\delta-12+24y-\cdot 45\xi+$,,	**0,**	–12	**–6,**	–3
18	20470	,, $+6\delta+76+24y-\cdot 45\xi+$,,	**–3,**	4	**3,**	–5
19	20500	,, $+7\frac{1}{2}\delta+71+26y-\cdot 48\xi+$,,	**–3,**	–7	**3,**	–4
20	20529	,, $+9\delta+80+26y-\cdot 48\xi+$,,	**–3,**	2	**3,**	5
21	20559	,, $+10\frac{1}{2}\delta+71+26y-\cdot 48\xi+$,,	**–3,**	–7	**3,**	–4
22	20581	,, $+11\frac{3}{4}\delta+10+26y-\cdot 51\xi+$,,	**0,**	10		
23	20596	,, $+12\frac{1}{2}\delta+11+26y+\cdot 53\xi+$,,	**0,**	11		
24	20636	,, $+14\frac{1}{2}\delta+36+26y-\cdot 62\xi+$,,	**–1,**	13	**4,**	–13

Of the above (14, 18) must be set aside at once: (14) because it belongs to the triplet set linked by $2e$ to the parallel set F (1), and (18) because it is F_2 (2). It may also be noted that neither have the prevalent separation 1780 to lines of higher frequency. Of the numbers on the right of the list, those in thick type give the values of y which bring the outstanding differences to the corresponding number in ordinary type. These differences must be due to errors either in ξ or observation. Since ξ is small, it is seen that they must be capable of annulment by the observation errors $6(q-p)$, and must, therefore, at the maximum be <12. The smallness of ξ can be seen from the following considerations which connect it with the corresponding ξ (say ξ') for the 1864 F series. The limit of the F is $30725\cdot 30+\xi'$ with mantissa $889322-30\cdot 74\xi'$. It is a d (1) sequent. If 19989 is a D line with $y=0$, its sequent is $51025\cdot 29+\xi'-19989\cdot 72=32035\cdot 57+\xi'$. The mantissa of this is $879853-30\cdot 29\xi'+6q$. Both being d sequents must differ by a multiple of the oun. Their difference is $9469+30\cdot 29\xi-30\cdot 74\xi'-6q$ and $15\frac{1}{2}\delta=9470$. Hence

$$30\cdot 29\xi-30\cdot 74\xi'=1+6q,$$

or,

$$\xi=1\cdot 015\xi'+\cdot 03+\cdot 2q.$$

Thus $\xi=\xi'\pm\cdot 3$. Since the 1864 limit is determined as the mean of F and **F** series its value is subject to a very small uncertainty and ξ' will be a small fraction.

Consequently in the above list the value of ξ has no importance in settling the order of the displacements.

The whole of the foregoing argument is based on the constancy of Δ_2 for all series. This matter has been referred to in the introduction in which the question of what is to be understood by the atomic weight was brought up. The accuracy in the determination of the oun is rendered so great by the constitution of the d and f sequences, that the mass of the electrons connected with the nucleus affect it. It may be interesting to illustrate the considerations there adduced by a concrete example. The example we will take is (1), as the result may possibly throw some light on a difficulty which will appear later. The line 19880 is seen to require a displacement of $-\delta$. Let us determine the transfer of electrons in order to produce a change by one oun. Suppose this transfer changes x to $x+x'$. This means that the mantissa as represented in the list must be diminished by $80x'$, by putting $y = -1$.

Hence

$$x' = \frac{52}{80} = \cdot 65, \qquad d\delta = \cdot 036.$$

The change in the number of electrons (see p. 342) $= 925 \times \cdot 036/611 \times 130 = 1 \cdot 97 = 2$. The addition of eight electrons to the mass acting in our standard case, would render 19980 a possible D line with limit $(-\delta_1)\,\mathrm{D}(\infty)$ instead of a possible one with $(-\delta)\,\mathrm{D}(\infty)$.

The preceding treatment of the material is only a first step towards unravelling the intricacies of these D systems. An exhaustive treatment is here impossible, and would involve the consideration of other data—the triplet separations, linkages, similarities of arrangement, dependence on F series and so on. All that can be done is to give a few illustrative instances and to bring into prominence certain problems whose solution in the future may be of extreme importance.

(1) The line 20021 is given as requiring the displacement $-3\delta_1$ in the limit, and in this case the mantissa is $80\Delta_2$. This displacement increases the limit 51025 by 31·88, and the resulting ν_1 should be greater by $3 \times \cdot 535 = 1 \cdot 60$ and $= 1779 \cdot 50$. As a fact the line (see p. 382) forms a doublet with separation 1779·51, and with intensities 1, 10, in the proper order for a D_{12} set. All the tests support each other. Again in the doublets

(7) 20500·13	**1784·68**	(2) 22284·82
(8) 20559·08	**1785·64**	($<1n$) 22344·72

the same displacements (or of $+3\delta_1$) are indicated. If we suppose these modified values of ν_1 produced in the same way as in Kr by a relative displacement in the sequent, the oun in this case alters the separation by 4·91. The modified ν_1 therefore becomes $1779 \cdot 50 + 4 \cdot 91 = 1784 \cdot 41$, or within error limits of the observed value for 20500, but too small for 20559. That for 20559 corresponds to an extra displacement of $-2\delta_1$ on the limit, or $-5\delta_1$ in all, which is quite inadmissible on the qualifying test.

The mantissæ of both lines differ by an exact 3δ. This is the natural conclusion, viz., same limit, sequents differ by 3δ, but then the value of 1785 remains unexplained, and we should not expect to find two D_{11} lines of the same group so close together. A possible explanation is to allot 20559 to the alternative displacement of $3\delta_1$ which gives $\nu_1 = 1776{\cdot}30$ and to take $2\delta_1$ in the sequent of the second line. This would give a modified separation $1776{\cdot}30+2\times4{\cdot}91 = 1786{\cdot}12$ or ·48 greater than that observed. It may be noted that as they stand the four lines (18 to 21 of list) requiring $-3\delta_1$ displacement have their sequence mantissæ equally spaced by $1\frac{1}{2}\delta$. The line 20041 also comes into the system with an exact δ sequence displacement from 20021.

In an analogous position to the 16013 line in the $D(\infty)$ set appears the $(-3\delta_1) D(\infty)$ set

[16044·05] **1785·37** (1) 17829·41 **810·13** (2) 18639·54

in which the 16044 is extrapolated from 18639 by the $\nu_1+\nu_2 = 2595{\cdot}5$. As the oun displacement on the sequent here produces 6·06, the modified ν_1 should be $1779{\cdot}50+6{\cdot}06 = 1785{\cdot}56$, and the modified $\nu_2 = 816{\cdot}02-6{\cdot}06 = 809{\cdot}96$ which agree with the observed. We may regard the 16044 as subject to the observation errors of 18639, say $d\lambda$. The mantissa of 16044 is $769863 = 70\ (10998{\cdot}04-{\cdot}36d\lambda)$, or $70\Delta_2$ as in the case of $D(\infty)$.

With the examples here given the arguments from the capability test and the observed separation agree. Complete sets for $(-3\delta_1)\ D(\infty)$ series are obtainable but are not here adduced.

(2) The line of 19942 has already been considered on (p. 396) as a $(4\delta_1)\ D(\infty)$. It again is a case where the capability test and ν_1 separation both point to the limit displacement of $4\delta_1$ or δ. The list also gives 19959 as requiring the same displacement. But it is $u.F_4(3)$ of the series below and shows a forward link 1864 in analogy with these F series.

(3) The line 20312 is clearly of special importance. It gives the source of the 1864 separations. As is seen the capability test reqires $-4\delta_1$ but there are many difficulties in the way of properly placing it. It forms part of a strong doublet (6) 20312 **1783·72** (5) 22096 in which the intensities are not in normal order. If 1783·72 is the usual modified ν_1, the sequence oun displacement is here 4·98 and the true $\nu_1 = 1783{\cdot}72-4{\cdot}178 = 1778{\cdot}74$. This differs only by ·13 from that proper to $(-2\delta_1)\ D(\infty)$, instead of to the $-4\delta_1$. The 1864 separations are greater than in the F series, being 1865·51, 832·13 against those found from the F above 1865·10, 829·64. The latter were based on the limit 30725·30 on which a displacement of δ_1 produces ·45 in 1864 and ·22 in 829. There is clearly here some triplet modification as the ratio of the separations is not correct. With 1865·51 should go 830·15 or with the given sum 1866·87, 830·77. Now $-3\delta_1$ on the limit produces 1865·45 and increase the limit itself to 30740·21. This should be the d sequent for 20312. If so the D limit should

be 30740·21+20312·70 = 51052·91 which is 27·62 above the normal and has no reference to the oun displacement. If 20312·70 corresponds to the $-4\delta_1$ indicated, the limit is 51067·79 and the d sequent is the difference, or 30755·09. This is the source of the 1864 separations. It is 22·79 greater than the limit 30725·30 of the F series considered in connection with this series and which gave 1864·10 and 829·64. This limit corresponds to a sequent $6\delta_1$ less displacement or 29·82 greater value which is practically exact. This present sequent will therefore increase 1864·10 by 6 × ·45 to 1866·70 and 829·64 by 6 × ·22 to 830·96. The new sum is 2697·66, as against the observed 2697·44 in remarkably close agreement. This then supports the $-4\delta_1$ indication of the capability test. A $-3\delta_1$ displacement would make the sum = 2697·00 —which though a worse agreement may yet be within observation errors. The same $-3\delta_1$ would make the modified ν_1 = 1779·50+4·97 = 1784·47, or ·72 greater than the observed. One is almost tempted to suspect here an error greater than the ordinary observation error. An error of ·36, $d\lambda = -·09$, would make all three tests agree in allotting 20312 to the $-3\delta_1$ set, and would bring it into the group of lines considered in Case 1.

It should be noted that this 20312 line is the line with wave-length 4922 referred to by LIVEING and DEWAR for its peculiar behaviour (see p. 350).

(4) The separations of the triplet 19880 are 1778·42, 815·30. They suggest the separation $-\delta_1$ to which belong 1778·43, 815·42. The capability test gives $-3\delta_1$ and the two are incompatible. In the D series given below the calculated limit from the first three lines give the limit as 51045·37 or 20·08 above the normal. With the uncertainty in a limit found in this way this is a displacement of $-2\delta_1$ which gives 21·25. Further there seems some evidence to show that this line with 20021, 20041, 20312, 20500, 20559 belong to one D (1) group of lines. The evidence consists in the existence of parallel F sets showing displacements equal to the separations of these lines. In other words these lines are D sets with the same limit. All these tests mutually exclude each other. How can their indications be reconciled? I suggest

(1) The limit for the line is $(-\delta_1)$ D (∞) and the observed ν_1, ν_2 are thus explained.
(2) The capability test is met by the transfer of six electrons indicated above.
(3) It is not the first line of the series in question.
(4) The F separations will be found to offer a natural explanation.

In what has preceded an attempt has been made to allocate normal D series, but they are clearly not the strongest sets. In my first attack on the D, F problem the procedure adopted was to take the 19880 triplet as a clear satellite set, and attempt by the application of RYDBERG's table to find a series for this satellite series. The three sets found were :—

(1) 19880	**1778·42**	(5) 21659·14	**815·30**	(10) 22474·44
[37888·59]	**1777·90**	(1) 39666·49	**814·42**	(2) 40480·91
[43801·13]	**1777·74**	[45578·87]	**818·18**	[46397·05]

The second set is near the end of the observed region, and the fainter D_1 line is extrapolated by the normal ν_1, this will introduce a small possible error. The third set is wholly outside the observed region and was obtained by sounding. The data are as follows, the lines used being regarded as linked by e to the first triplet 36487, ...

[43801·13]	$d\lambda$	[45578·87]	$d\lambda$	[46397·05]	
(1) 36487·03.e	·00	(1n) 38264·77.e	·00	(1) 39082·95.e	·00
(1) 29175·18$e.e$ (δ_1)	·00	(1) 30949·88.2e	·08	(<1) 31768·93.2e	−·01
(1) 39372·71.v	−·02	(1) 41155·14 v $(2\delta_1)$	·03	(<1) 34659·40.$e.v$ $(2\delta_1)$	−·07
(1) 39666·49.u	·09	(1) 33841·74.$e.v$ $(2\delta_1)$*	·12		
(3) 32351·67.$e.u$ $(-\delta_1)$	·10	(1) 34126·78.$e.u$ $(-2\delta_1)$*	−·12		
(<1) 24749·35.$e.e$ (δ_1).v (δ_1)	−·04	(1) 26814·18.2$e.u$ $(2\delta_1)$	·03		

The formula obtained from these is

$$n = 51045{\cdot}37 - \mathrm{N}\Big/\left\{m + {\cdot}898460 - \frac{{\cdot}022504}{m}\right\}^2.$$

This was tested to $m = 13$, with good agreement with the exception of $m = 4$. All are in the ultra-violet and require sounding. As the evidence of the efficacy of sounding already adduced may be regarded as convincing, there is no advantage in giving further details of the results, especially as no additional conclusions are based on this formula.

XF.—We are to look for parallel F sets with the same separations as those of the D satellites. Starting with the triplet 19880 as an undoubted D satellite triplet it was then attempted to determine the others, by picking out those lines of larger wave-length whose mantissæ differed from that of 19880 by oun multiples. The data are contained implicitly in the list on p. 407, independently of the form into which the mantissæ are there thrown. It is clear that the above condition is satisfied by all those which show unsatisfied remainders of the same magnitude as that of 19880—*i.e.*, 66—within error limits of, say, ± 6. The selected lines were 19880, 20021, 20041, 20312, 20500, 20559. We have shown above that the second, third, fifth and sixth of these satisfy the capability test for $-3\delta_1$ displacements on the limits, and have also found some evidence that 20312 also belongs to this set, although perhaps with a somewhat excessive observation error. It has also appeared that 19880 cannot belong to this set, but it has so happened that the satellite separations from it reproduce themselves in the F series sufficiently well as to serve for identification. We shall, therefore, use this allocation. The selected lines give separations from 19880 respectively of 140·94, 161·01, 431·98, 619·41, 679·36. In these 19880 was treated as if it were a fifth satellite to 20559 and the notation adopted of F_6, ... F_1 series. Such series were found, and in what follows I shall chiefly confine consideration to them leaving aside for the present purpose, with one or two exceptions, the very numerous other groups which exist. It would not be advisable to suggest a definite notation

* Parallel inequality.

for these groups until the whole system of D and F series is placed on a secure and comprehensive foundation. The above notation, therefore, is only to be regarded as one which in this communication serves to identify certain of those groups here more specially discussed.

The material was treated by the same method as in Kr, and search made for the above separations in the region where F (2) lines should be expected. From the lines thus obtained the actual F lines were sought for, bearing in mind the large sequence displacements so common in the low orders. In only three cases—those for $F_{2, 3, 6}$—were suitable undisplaced lines found for $m = 2$, and of these only F_3, F_6 gave observed lines for $m = 3$. They were—separations from F_6—

m.	F_6.		F_3.		F_2.
2.	(8) 18812·55	**432·46**	(8) 18380·09	**619·82**	(1) 18192·73
3.	(1) 24253·30	**432·49**	(<1) 23820·81		
5.	(2) 28108·52	**432·00**	(1) 127676·52		

For $m = 4$, F_3 showed no undisplaced line, but in good position for $m = 5$ there were lines for F_6, F_3. From the three lines for F_3 the following formula was found

$$30740{\cdot}17+\xi-\mathrm{N}\Big/\left\{m+{\cdot}986181-622\xi-\frac{{\cdot}014730-1003\xi}{m}\right\}^2.$$

We can at once apply two tests to this as to fulfilling the conditions for F series. The limit 30740 must be a d-sequent, *i.e.*, must differ by oun multiple displacement from some other known d-sequent, and the first f-sequent must depend directly on a multiple of Δ_2. We already have a very accurate d-sequent found as the limit of the 1864 series, viz., $30725{\cdot}30+\xi'$. On this a δ_1 displacement produces a change of 4·97 so that $-3\delta_1$ produces $30725{\cdot}30+\xi'+3\times 4{\cdot}97 = 30740{\cdot}21+\xi'$, and this condition is accurately satisfied by $\xi_1 = \xi'+{\cdot}04$ where ξ_1 belongs to the present case. For the second test the mantissa of $f(2)$ is

$$978816-121\xi_1 = 89\,(10997{\cdot}93-1{\cdot}36\xi_1) = 89\,(\Delta_2-{\cdot}27-{\cdot}36\xi_1-x)$$

which again is seen to satisfy the test exactly. The satisfaction of these two conditions must give full confidence as to the correctness of the allocation of the F_3 lines.

From this formula lines were calculated from $m = 4$ to 27 and the other sets allocated by their corresponding separations. They are not reproduced here, however, beyond $m = 15$, partly because they are only of importance in a systematic arrangement of the X spectrum, and partly because the lines to be identified are so close that it becomes a matter of extreme difficulty to allocate them correctly. For instance, the calculated values of F_2 (14) and F_1 (15) are 30064·38 and 64·16; of F_4(12) and F_3(16) are 30360·47 and 60·29 with many other examples. With the high orders successive lines become close, and with the large number of separate series involved the observed spectrum should be expected to be crowded, as indeed in this region it is.

The results are given in the annexed table, with notes up to $m = 7$. Numbers in brackets refer to values obtained—when none have been directly observed—by connection with linkages or displacement. It may be recalled that a link from an observed line to an expected but unseen one leads to the inference that the unobserved really exists, whereas a displaced line, when the displacement is on the limit, gives evidence only for the value of the sequent $f(m)$, and when the displacement is on the f-sequent, is evidence to that effect alone.

The true $F_3(\infty)$ is $30740{\cdot}17+\xi$ where ξ is small. The other limits will depend on oun displacements from this. Estimated from 20312—the source of $F_3(\infty)$—these displacements, expressed as multiples of δ_1 are $-21\frac{1}{2}$, $-14\frac{1}{2}$, $-13\frac{1}{2}$, $9\frac{1}{2}$, $12\frac{1}{2}$. Their values can, therefore, be calculated with exactness relatively to $F_3(\infty)$. They are

F_1.	F_2.	F_3.	F_4.	F_5.	F_6.
$30493{\cdot}11+1{\cdot}012\xi$	$30552{\cdot}13+1{\cdot}01\xi$	$30740{\cdot}17+\xi$	$31010{\cdot}41+{\cdot}99\xi$	$31030{\cdot}58+{\cdot}98\xi$	$31172{\cdot}24+{\cdot}98\xi$
4·910	4·927	4·964	5·030	5·043	5·090
	59·02	247·06	517·30	537·47	679·13

The numbers below the limits give respectively the changes produced in them by the displacement of one oun. The numbers in the last line give the calculated accurate separations of the corresponding F lines from F_1.

For the first order, $m = 2$, considerable displacements are to be expected. Only normal lines for $F_{2,\,3,\,6}$ are observed. The set (5) 18998·40, (3) 19515·81, (7) 19676·25 give close normal separations 517·41, 677·85. Now the limit of F_1 is 30493·11 and the denominator of 18998 calculated from this is $3{\cdot}088906-134{\cdot}3\xi$, or a mantissa $= 1088906-134{\cdot}3\xi+24p = 99\,(10998{\cdot}14-1{\cdot}35\xi+{\cdot}24p) = 99\Delta_2$. The normal $f(2)$ sequent is $89\Delta_2$. There is, therefore, a displacement of $10\Delta_2$ in the sequence term. Further the defect in the separation 677·85 from the normal is 1·28 whilst a δ_1 displacement on the sequent produces 1·11. The lines in question, therefore, are $F_1(2)\,(10\Delta_2)$, $F_4(2)\,(10\Delta_2)$, $F_6(2)\,(10\Delta_2-\delta_1)$.

To find a representative for F_5 we may test (1) 18332·41, **290·35**, (3) 18622·76 and (1) 18018·31, **289·60**, (6) 18307·91 in which the separations refer to that of F_3, F_5, viz., 290·40. The mantissæ difference for the first set is 5740, and the nearest oun multiple $9\frac{1}{2}\delta = 5804$ is outside error limits. That of the second is 42758 and $4\Delta_2-2\delta = 70\delta = 42770$. An observation error of $d\lambda = {\cdot}03$ would make this exact. The lines in question may therefore be $F_3(2)\,(70\delta)$, $F_5(2)\,(70\delta)$. So also it may be shown that 18466·47 is $F_2(2)\,(-2\Delta_2)$.

With the F difference-series occur also the **F** summation type. As their existence is a new fact of great importance the evidence available up to $m = 10$ is given. The results are embodied with those of F in the table.

TABLE of the $F_n.F_n$ Lines

	n = 1.	ν_2.	n = 2.	ν_3.	n = 3.
Limit	30493·11	**59·02**	30552·13	**247·06**	30740·17
m. 2	[18133·42] **93·63** (42853·85) (1n) 2581.u (5) 5262 18998·40	**59·31** **57·68** } F (10Δ_2)	(1) 5495 18192·73 **52·13** (42911·53) (δ_1) (1) 2808.e	**246·67** **247·91**	(8) 5439 18380·09 **40·92** (43101·76) ? (<1) 3187.$e.v$
3	(3) 4240 23576·11 **92·72** 37409·33 (3) 2672·35	**59·21** **59·01**	v.(3) 3562 (23635·32) **51·83** 37468·34 (3) 2668·14	**244·70** **251·29**	(<1) 4196 23820·81 **40·74** (37660·62) ($-3\delta_1$) (1) 2655·57
4	(2n) 4617.v (26078·04) **92·12** 34906·21 (<1) 2864	**59·25** **57·98**	($-2\delta_1$) (1) 3826 (26137·29) **50·74** (34964·19) ($3\delta_1$) (1) 2858	**247·86** **247·92**	v.(3) 3250 (26325·90) **40·01** (35154·13) ($-2\delta_1$) (3) 2844
5	(2) 3644 27430·33 **91·83** 33553·33 (6) 2979	**58·49** **60·86**	($-\delta_1$) (2) 3636 (27488·82) **51·50** (33614·19) (δ_1) (3) 2973	**246·19** **251·18**	(1) 3612 27676·52 **40·51** (33804·51) ($-\delta_1$) (5) 2957
6	(1) 4197.v (28242·73) **94·09** (32745·46) (1) 3530·40.v	**58·23** **59·26**	(δ_1) 3531·93 (28300·96) **52·49** (32804·72) ($-\delta_1$) (1n) 3047	**247·06** **246·90**	(1) 3509 28489·79 **41·07** (32992·35) ($-\delta$) (1n) 3031
7	(1) 3474 28773·67 **93·06** (32212·45)	**58·57** **59·96**	(5) 3467 28832·24 **52·33** (32272·41)† ($-2\delta_1$) (1) 3098	**245·65** **247·29**	(1) 3445 29019·32 **39·53** 32459·74 (4) 3079

* F_6 (7) *and* F_2 (9) *are coincident.*

from $n = 1$ to $n = 6$.

ν_3.	$n = 4$.	ν_5.	$n = 5$.	ν_6.	$n = 6$.
517·30	31010·41	**537·47**	31030·58	**679·13**	31172·24
517·86 **517·41**	(43371·71) (4) 2772.*e* (3) 5122 } 19515·81 }	**536·27**	(43390·12) (1*n*) 2546.*u*	**679·13** **678·29** **677·85**	(8) 5314·15 18812·55 **72·34** (43532·14) (δ_1) (<1) 2759.*e* (7) 5080 } 19676·25 }
516·71 **519·02**	(1) 5008.*u* (24092·82) **10·58** 37928·35 (1) 2635	**537·71** **538·89**	(5) 4145 24113·82 **31·52** 37949·22 (3) 2634	**677·19** **679·81**	(1) 4122 24253·30 **71·22** 38089·14 (1) 2624
517·35 **515·84**	*u*.(5) 3253 (26595·39) **08·22** 35421·05 (1) 2822	**537·59**	(1) 3756 26615·63	**680·03** **680·47**	(1) 3736 26758·07 **72·37** 35586·68 (3) 2809
519·81 **517·53**	(5) 3576 27950·14 **10·50** (34070·86) ($3\delta_1$) (4) 2932·72	**537·32** **540·17**	(1) 3574 27967·65 **30·57** 34093·50 (3) 2932·27	**678·19** **682·82**	(2) 3556 28108·52 **72·33** 34236·15 (3) 2920
517·55 **514·93**	(δ_1) (1) 3475 (28760·28) **10·33** (33260·39) ($2\delta_1$) (1) 3004·81	**537·14** **535·50**	(<1*n*) 4056.*u* (28779·87) **30·41** 33380·96 (2) 2994	**678·89** **676·36**	(<1) 4033.*u* (28921·62) **71·78** (33421·95) (δ_1) (1) 2990·74
514·91 **520·56**	(1*n*) 3413 29288·58 **10·79** (32733·01) ($-\delta_1$) (4) 3054	**536·94** **537·63**	($2\delta_1$) (<1*n*) 3409 (29310·61) **30·34** (32750·08) ($-2\delta_1$) (2) 3494.*u*	**678·95** **679·90**	($-\delta_1$) (2) 3394 (29452·62)* **72·48** (32892·35) (δ) (1) 3037

† Coincident values.

TABLE of the $F_n.\mathbf{F}_n$ Lines

	$n = 1.$	$\nu_2.$	$n = 2.$	$\nu_3.$	$n = 3.$
m. 8	(1*n*) 3998.*u* 29134·56 **92·83** (31851·09) $(-\delta_1)$ (1) 3139‡	**58·74** **59·46**	(1) 4569.*e* 29193·30 **51·92** 31910·55 (1) 3132	**246·56** **247·11**	*e.*(2) 3070.*u* 29381·12 **39·66** 32098·20 (4) 3114
9	$(2\delta_1)$ (1) 3400 (29393·35) **92·82** 31592·28 (1) 3164·43	**59·15** **59·01**	$(-\delta_1)$ (2) 3394 (29452·50)* **51·90** (31651·29) (δ_1) (4) 3632.*u*	**247·58** **248·87**	*e.*(δ_1) 3045.*u* (29640·93) **41·04** (31841·15) δ_1 (1) 3139‡
10	(2) 3379 29584·48 **92·58** (31400·68) $(-2\delta_1)$ (3) 3184·74	**60·21** **59·04**	(3) 3918.*u* (29644·69) **52·20** (31459·72) (δ_1) (2) 3177	**248·12** **247·61**	(δ_1) (1*n*) 3350 (29832·60) **40·44** (31648·29) $(-2\delta_1)$ (2) 3160
11	(1) 3362 29727·58	**59·41**	(1) 3942.*v* (29786·96)	**247·35**	(δ) (5) 3332** (29974·93)
12	(3) 3349 29843·17	**58·88**	*v.*(5) 2912 (29902·05)	**246·94**	*v.*(<1) 2896 (30090·11)
13 F $(2\Delta_2)$ or	(1) 3339 29934·65 (δ_1) (1) 3339·37 29932·42	**60·16** **59·57**	(5) 3332 29994·81 $(-2\delta_1)$ (1) 3334 29991·99	**247·01** **247·34**	(<1) 3312 30181·66 $(-2\delta_1)$ (1*n*) 3313 30179·76
14	(5) 3331 30005·34		[30064·38]	**247·17**	$(2\delta_1)$ (2) 3303 30252·51
15	[30064·16]	**59·04**	(1) 3318†† 30123·20	**248·14**	(<1) 3298 30312·30

* $F_6(7)$ *and* $F_2(9)$ *are coincident.*

† Coincident values.

‡ May be either.

§ *Coincident with* $(-2\delta_1)$ $F_1(12)$ *of* 1864 *series.*

from $n = 1$ to $n = 6$ (continued).

ν_3.	$n = 4$.	ν_5.	$n = 5$.	ν_6.	$n = 6$.
517·37 **516·92**	[29651·93] **09·97** (32368·01) $(-\delta_1)$ (1) 3089	**536·65** **537·09**	[29671·21] **29·70** (32388·18) (δ_1) (3) 3537.*u*	**677·80** **689·72**	$(-2\delta_1)$ (1) 3354 (29812·36) **71·58** (32530·81) $(-\delta_1)$ (4) 3073
516·71 **518·30**	*v*.(<1) 2911·38 (29910·06) **10·32** (32110·58) (δ_1) (3) 3112	**537·23** **538·45**	$(2\delta_1)$ (2) 3339·00 (29930·58) **30·65** (32130·73) $(-3\delta_1)$ (3) 3112	**679·52** **680·55**	*u*.(<1) 2922·62 (30072·87) **72·87** (32272·83)† $(-2\delta_1)$ (1) 3098
516·70 **519·93**	$(2\delta_1)$ (6) 3322§ (30101·18) **10·90** (31920·61) $(-2\delta_1)$ (1) 3132¶	**538·72** **537·00**	(1) 3318‖ 30123·20 **30·44** (31937·68) (2) 3595.*u*	**678·20** **680·16**	(2) 3303‖ 30262·68 **71·76** (32080·84) (δ_1) (1) 3614.*v*
516·61	$(-\delta_1)$ (4) 3306 (30244·19)	**535·10**	(2) 3303†† 30262·68	**677·17**	(5) 3288 30404·75
517·73	*v*.(2) 2873 (30360·90)	**539·32**	(<1) 3290 30382·49	**678·26**	$(-\delta_1)$ (3) 3276 (30521·43)
516·79 **516·98**	$(3\delta_1)$ (1) 3281 (30451·44) $(-\delta_1)$ (<1*n*) 3283 30449·40	**538·38** **538·01**	(4) 3280·66 30473·03 (<1) 3280·94 30470·43	**678·54** **678·89**	*v*.(δ_1) (2*n*) 2852 30613·19 $(2\delta_1)$ (4) 3264 30611·31
	[30522·68]	**538·53**	(3) 3273 30543·87	**679·30**	(1) 3258 30684·64
517·33	$(-\delta_1)$ (<1) 3269 (30581·49)	**537·17**	(δ) (4) 3264 (30601·33)	**677·56**	$(2\delta_1)$ (3) 3250 30743·72

‖ F_6 (10) and F_5 (11) are coincident.
¶ This line is F_2 (8).
** 3332 is F_2 (13).
†† F_5 (10) and F_2 (15) coincident.

In this table under each order the first line gives the wave-length of the observed line to the last Ångstrom, its intensity, and, where necessary, the displacement or linkage to be applied. The second line gives the wave-numbers of the F lines and the thick type the separations from the F_1 line adopted. Up to $m = 10$ the fourth line gives in the same way the wave-numbers of the **F** lines and the fifth the corresponding wave-lengths. In the third line the numbers give the mean limit $\frac{1}{2}(F+\mathbf{F})$, but only the last four significant figures are entered, the complete calculated values being given at the head of the table.

Notes to Table.—$m = 2$. For $\mathbf{F}_3$ in addition to that given there are (-2δ) (1) 38632·72.v = 43101·20, $(3\delta_1)$ (1) 38687·71.v = 01·53, (δ) (1) 38988·33.u = 01·27, (δ_1) (<1) 38973·59.u = 01·71.

$m = 3$. The linked F_2 agrees with $(3\delta_1)$ (2) 23650·79 = ...35·79. The linked F_4 with $(3\delta_1)$ (1) 24077·41 and $(-6\delta_1)$ (1) 24062·12 both of which give the same value ...92·30.

$m = 4$. The linked F_1 agrees with $(-6\delta_1)$ (1) 26048·86 = ...78·32. For F_2, $(2\delta_1)$ (1) 26147·89 = ...38·06 is closer to the calculated value ...37·95. F_4 has a link v = 4428·62. For F_6, $(-2\delta_1)$ (3) 26744·54 = ...54·72 is only ·26 greater than the calculated value. Most of the observed lines of this order are one or two units larger than the calculated. $\mathbf{F}_3$ is also given by $(3\delta_1)$ (<1) 37675·75 = 60·66. For $\mathbf{F}_4$, $(-\delta_1)$ (2) 35417·16 = ...22·19 gives separation correct. For $\mathbf{F}_6$ $(-\delta_1)$ (1) 35580·98 = ...86·07 gives much closer separation.

$m = 5$. For F_2, $(2\delta_1)$ 27498·89 = ...88·03. For F_4, e.35261·18 = 27947·18 and $(3\delta_1)$ 27963·59 = ...48·50 both give better separations. F_5 shows a series inequality with $-(u+3\cdot28)$ and $u-2\cdot93$. For F_6 also (δ_1) 28113·58 = ...08·49 and (δ) 28133·98 = ...13·62. $\mathbf{F}_1$ is a strong observed line which makes the mean limit ...91·83 too small and some of the other separations too large. $(-2\delta_1)$ 33544·55 = ...54·37 or $(-5\delta_1)$ 33530·95 = ...55·50 are better. The latter makes mean limit ...92·92 practically exact, and the separations 58·70, 249·00, 538, 680·65 all much improved.

$m = 6$. All the F are in good agreement with the calculated except for F_2. For this e.35617·10 = 28303·00, but too large. Also 28335·60, 28305·20 differ by 30·40 and $6\delta_1$ gives 29·58. Near $\mathbf{F}_1$ 32727·98, 32762·39 differ by 34·41 and $7\delta_1$ gives 34·37.

$-3\delta_1$ on the first or δ on the second give 32742·71 a better line for $\mathbf{F}$ as it makes the limit sum = 92·77 and gives better separations with $F_{3,4,5}$. There are clearly two sets with probable displacement in the f sequent. With the $\mathbf{F}_1$ in the table would go better $(-6\delta_1)$ 33232·22 = ...62·39 for $\mathbf{F}_4$ and $(-\delta_1)$ 33378·17 = ...83·21 for $\mathbf{F}_5$. The linked $\mathbf{F}_6$ agrees with (δ_1) 33427·04 = ...21·95.

$m = 7$. This presents several interesting points bearing on general theory. We may consider F_3 as correctly allocated since it differs only ·55 ($d\lambda$ = ·06) from the value calculated from the formula, but it is coincident with F_1 (7) of the 1864 series. Judging from the separations which are too small (except F_6) the observed F_1 is from 1 to 2 too large. This F = 28773·67 would seem to give some insight into the connection between sequent displacements and concomitant limit displacements or linkage attachments. Thus this line has relations with displaced limits with the two lines 28788·83 = $(-3\delta_1)$ F + ·43 and 28733·40 = (2δ) F − ·99 very close, but scarcely sufficiently so to exclude the probability of small sequent displacements. Further, it is linked forwards and backwards with all the three links e $u.v$ as shown in the following scheme on the left. The 24638, 32912 form with F_1 an exact series inequality. Now a series inequality indicates that in the successive lines each is displaced from the preceding by the same amount, in this case about $15\delta_1$. The whole set may then be arranged as indicated in the right-hand scheme where X = 28771·80 = 28773·67 $(-k)$ and k denotes the displacement (? $15\delta_1$). The k may be the same for the different links within observation errors,* but probably not. If we take X as normal F_1 the other

* To make exact it would require the following :—7δ gives 1·84 ; 4δ, 1·05 ; 8δ, 2·10 ; 9δ, 2·36 ; 3δ, ·79.

separations become normal also, but as they stand F_1 and $\mathbf{F}$ give the true mean limit. In other words they have the same sequent, or the sequent is modified in the same way for both. The normal $\mathbf{F}_1$ should be as much greater as the normal F_1 is smaller, and we do find such indications. The value given for $\mathbf{F}_1$ was found as the mean of two observed lines 32211·10 and 32213·80, supposed as $\pm$ displacements from the true line. If 32213·80 be taken as normal $\mathbf{F}_1$ it gives as mean limit with X ...92·80. The whole set of F and $\mathbf{F}$ in this order affords good illustrations of the remarks on $\boldsymbol{p}$ as to the general properties of these series. All the F lines show link connections. Thus F_2, $-(u-·1)$, $u-·1$; F_3, $+u-·73$; F_4, $-(u-·32)$, $u-2·33$, $v-·9$; F_5, $+v+1·7$; F_6, $+u+·2$.

	21456, 21462			e.X, e.X(2k)	
	e − ·29 ± 2·38				
24638		32912	u.X		X(2k).u
u + 1·87		**u + 1·86**			
	F_1			X(k)	
v − ·70		**v + 2·06**			
24346		33203	v.X(k)		X(2k).v
	e − 44·0				
	36083			X(−k).e	

In the preceding table the wave-numbers where deduced are entered as depending on one given line. But in nearly all cases these are substantiated also by links or other displaced limit lines. Although in an exhaustive treatment of the spectrum all these must be considered, this is not required for the illustration of general theory, and the notes do not go beyond $m = 7$. The number of the associated lines is remarkable. The proof of the existence of these lines depending on other d sequents as limits is so important, as a matter of general theory, that a large number have been adduced up to $m = 7$. To go beyond would be to overload the present communication with detail. The prevalence of the $u.v.$ links over e may be noted in the F series.

In dealing with the 1864 series the mean of the common F and $\mathbf{F}$ limits for different orders was treated as giving the true value within a very small possible error of about $\xi = ·3$. It may be interesting to see how in the present cases the corresponding averages deviate from the values which have been established on the 1864 basis. The mean values found from the tables and expressed in terms of differences from the adopted ones are ·26, ·24, −·33, ·06, ·05, ·15 and justify the supposition of the small limit of error in ξ. The mean separations as found from the same 10 F lines deviate from the correct values by ·02, ·86, ·05, ·06, ·11. The large deviation ·86 is due chiefly to the uncertainty of the displacement in $F_3(3)$.

The preceding discussion is sufficient to show the excessive number and complication of the lines belonging to the F systems in this neighbourhood and that the complete problem has only been touched upon. As however the allocation of the extrapolated lines 16013 and 16044 as extreme satellites of D series based respectively on $D(\infty) = S(\infty)$ and $(-3\delta_1)S(\infty)$ is important, it will be well to consider shortly if corresponding F series can be indicated. The two lines referred to give the same d sequence, depending on a denominator $1+70\Delta_2$. This makes $d(1) = 35012·30$.

Taking this as F (∞) and using the $f(m)$ sequences as given by the formula (p. 412) it is possible to calculate the positions of the lines in question. The results are given down to m = 8.

m.	F.		35012·30.	**F.**	
	Calculated.	Observed.		Observed.	Calculated.
2	22652·26	...(3) 52·88	11·98	(...71·08)	47372·34
3	28092·94	(...93·54)	11·85	(...30·16)	41931·66
4	30594·43	...(1) 95·63	13·50	(...31·36)	39430·17
5	31948·38	(...47·00)	12·02	(...77·05)	38076·22
6	32763·63	...(1) 62·39	12·76	...(1*n*) 63·13	37260·97
7	32292·00	(...92·05)	12·14	(.. 32·23)	36732·00
8	33653·80	...(3) 53·04			36370·80

[*Note.*—δ_1 displacement on F (∞) gives a change 6·045.]

2. **F** requires sounding. (2) 35923·80.*e.v* = 47371·08.
3. $(-5\delta_1)$ (3) 28063·32 = ...93·54; **F** requires sounding. (δ_1) (1) 37508·25.*v* = 41930·21.
4. $(-2\delta_1)$ (2) 39419·27 = ...31·36.
5. $(-2\delta_1)$ (2*n*) 31934·91 = ...47·00; $(2\delta_1)$ (1) 38089·14 = ...77·05.
6. For **F**. (1*n*) 37263·13; also $(-6\delta_1)$ (<1) 37226·34 = ...62·88, or the same.
7. *v*.(1) 37720·05 = ...92·05; also $(-3\delta_1)$ (4) 33274·11 = ...92·24; (2) 32304·23.*v* = 36732·23.

The mean limit is 35012·37. The set form an additional test that the extrapolated lines 16013 really exist. It is curious to note that the even orders of F only show directly observed lines, whilst the odd show displaced limit lines. The **F** lines are far to the violet end and come into the observed region only when weakened by high order. The F lines are all linked to lines of higher frequency by the 1864 link, also for m = 4, 6 to lines of lower frequency. The same tendency is shown in **F** to lower frequency, any such to higher frequency lead to unobserved regions. This fact is important as showing that at least here the 1864 separation enters in the link relation, and not as a direct displacement on the limit.

To the $F_3(\infty)$ limit corresponds a 1864 triplet series parallel to that originally considered. I have been able to follow it up in the same way as the foregoing as far as m = 26 at least. It accentuates the evidence for the displaced sets but as that is sufficiently supported by the results already discussed it would seem unnecessary to overburden the present communication with additional detail. Whenever the actual relations of the various displaced lines to one another are the subject of discussion these details will be of the first importance. A knowledge of these relations should be expected to throw a flood of light on the constitution of spectra, but this new question cannot be taken up here. It may be noted, however, that the first F and **F** lines of the triplets up to m = 10 are given in the table under F_3.

RITZ *Combinations.*—The results obtained enable us to test for additional series, associated with the name of RITZ who first pointed out their existence. They are represented by the expressions $n = p(1) - f(m)$, and $s(1) - f(m)$. Here $p(1) = \mathrm{S}(\infty) = 51025{\cdot}30$ and $f(m)$ is given by the formula on (p. 412). $s(1) = \mathrm{P}(\infty) = 93178{\cdot}69$ and should give a parallel series 42153·39 ahead, and therefore in the extreme ultra-violet. The results are for the sets p_1, p_2, p_3.

m.	Calc. $p_1 - f(m)$.	$d\lambda$.	Observed.				
2.	38665·21	−·07	(1) 38666·56	**1784·06**	(2) 40450·62	**806·91**	(1) 41257·53
3.	44105·93	·18	(44102·34)	**1783·08**	(45885·42)	**815·66**	(46701·08)
4.	46607·42	·03	(46606·52)	**1784·28**	(48390·80)	**807·29**	(49198·09)
5.	47961·81	·06	(47960·42)	**1777·90**	+	**813·68**	(50552·00)

Notes.—$m = 2$. 1784·06+806·91 = 1777·90+813·07

$m = 3$. There is some ambiguity as to $\mathrm{F}_3(3)$, which gives the calculated value. The deduced ultra-violet lines are from the observed triplet by the e link.

(7) 36788·24 **1783·08** (2) 38571·32 **805·66** (1) 39386·98.

Also

$e+v+$(1) 32362·98 **1777·90** (<1) 34140·88 give 44105·08, 45882·98.

$u+$(2) 39969·62 = 44102·8 with the ν_1, ν_2 lines in the ultra-violet, but an extra v sounder gives $u+v+$(2) 37320·68 **813·93** (1) 38134·61 = 45881·86, 46695·79, the former being 1779·06 ahead of 44102·80.

$m = 4$. $e+$(1) 39292·42 **1784·28** (1) 41076·70 = 46606·52, 48390·80 with $2e+$(4) 34569·89 = 49198·09 which is 1777·90+813·67 above 40606.

$m = 5$. $2e+(2u)$ 33332·22 **1779·90+813·68** (2) 35923·80 = 47960·42, 50552·00; also $e+u+$(1) 38292·32 **815·34** (1) 39107·56 = 49739·60, 50554·94; the former being 1777·79 above the calculated first line of 47961·81.

The Value of the Xenon Oun.—In the case of xenon the triplet disturbance appears to be small. The result has been obtained that $\Delta_1 = 24893 = 40\tfrac{3}{4}\delta$ and $\Delta_2 = 10999 = 18\delta$, giving respectively for δ the values 610·87, 611·05 or 610·94 from $\Delta_1 + \Delta_2$. In the preceding 611 has been adopted for δ which is sufficiently close except where very large multiples of δ are in question. To obtain more accurate values of Δ_2 recourse must be had to the F and D sequences. Incidentally they have already been touched upon, but we are now in a position by a discussion of the whole material to attain a greater exactness.

Xenon, as has been seen, shows numbers of parallel displaced groups of D and F series, each of which gives data for the determination of Δ_2. Unfortunately those lines which might be expected to give the most accurate values are in the ultra-red beyond the observed region, and the process of extrapolation by the ν_1 separation, or by links, leaves a considerable margin of uncertainty owing to the 1780 modification of ν_1, and the well established, but not yet thoroughly understood, modifications of the e links, although in X this latter is not so marked as in Ag and Au. The material at disposal is—

From the D *Series.*

1. $[16013{\cdot}45] = 17791{\cdot}35-\nu_1 = 51025{\cdot}29+\xi-d(1+70\Delta_2)$ . . . (p. 404),
2. $[16044{\cdot}24] = 17829{\cdot}41-\nu_1-\nu_2 = (-3\delta_1)\ 51025{\cdot}29+\xi-d(1+70\Delta_2)$ (p. 409),
3. $19602{\cdot}66 = 51025{\cdot}29+\xi-d(1+79\Delta_2-\delta)$ (p. 403),
4. $[19623{\cdot}05] = 21400{\cdot}95-\nu_1 =$ „ $-d(1+79\Delta_2)$. (p. 403),
5. $19942{\cdot}53 = (\delta)($ „ $)-d(1+80\Delta_2-\delta_1)$ (p. 396),
6. $19989{\cdot}72 = ($ „ $)-d(1+80\Delta_2)$. (p. 396),
7. $20021{\cdot}66 = (-3\delta_1)($ „ $)-d(1+80\Delta_2)$. (p. 396).

From the F *Series.*

8. $[3010{\cdot}35] = 17638{\cdot}55-2e = 30725{\cdot}26+\xi'-f(1+90\Delta_2)$ . . . (p. 388),
9. $18380{\cdot}09 = F_3(2) = 30740{\cdot}17+\xi_1-f(2+89\Delta_2)$ . . . (p. 412),
10. $30725{\cdot}26+\xi' = d(1)$ sequent.

The relations between the ξ, ξ', ξ_1 have been already determined. $\xi = 1{\cdot}015\xi'+{\cdot}03+{\cdot}2q$ from the fact that (10) and the sequents of (6) are both d sequents (p. 407). Here q is the proportion of maximum error in (6). The ·03 may be supposed merged in this and $\xi = 1{\cdot}015\xi'$. Also $\xi_1 = \xi'+{\cdot}04$ from the fact that (10) and $30740{\cdot}17+\xi_1$ are both d sequences (p. 412). Within the accuracy attainable for our present purpose we may treat the ξ's as all equal. This was certainly not to be expected, for the limit S (∞), obtained by formula constants from S lines, is in general uncertain to a few units, whilst the ξ_1 and ξ', depending on limits found from $\frac{1}{2}$(F and **F**) should be very small.

The mantissæ conditions of the above give ($d\lambda = {\cdot}04p$)

*1.	$769890+\ 2{\cdot}5p_1-25{\cdot}27\xi = 70\Delta_2$	$\Delta_2 = 10998{\cdot}43-\ {\cdot}36\xi+{\cdot}036p_1$
*2.	$769867+\ 2{\cdot}5p_2-25{\cdot}27\xi = 70\Delta_2$	$= 10998{\cdot}10-\ {\cdot}36\xi+{\cdot}036p_2$
3.	$868240+\ 4{\cdot}6p_3-29{\cdot}73\xi = 79\Delta_2-\delta$	$= 10998{\cdot}16-\ {\cdot}37\xi+{\cdot}06p_3$
*4.	$868846+\ 5{\cdot}5p_4-29{\cdot}75\xi = 79\Delta_2$	$= 10998{\cdot}05-\ {\cdot}37\xi+{\cdot}07p_4$
5.	$879711+\ 4{\cdot}8p_5-30{\cdot}29\xi = 80\Delta_2-\delta_1$	$= 10998{\cdot}29-\ {\cdot}37\xi+{\cdot}06p_5$
6.	$879853+\ 4{\cdot}8p_6-30{\cdot}29\xi = 80\Delta_2$	$= 10998{\cdot}16-\ {\cdot}37\xi+{\cdot}06p_6$
7.	$879855+\ 4{\cdot}8p_7-30{\cdot}29\xi = 80\Delta_2$	$= 10998{\cdot}18-\ {\cdot}37\xi+{\cdot}06p_7$
*8.	$989285+\ 35dn-35{\cdot}4\xi = 90\Delta_2-\delta$	$= 10998{\cdot}84-\ {\cdot}39\xi+{\cdot}41dn$
9.	$978842+16{\cdot}4p_9-120\xi = 89\Delta_2$	$= 10998{\cdot}22-1{\cdot}35\xi+{\cdot}18p_9$
10.	$889326+\ -30{\cdot}28\xi = 80\Delta_2+15\frac{1}{2}\delta$	$= 10998{\cdot}19-\ {\cdot}37\xi$

leaving out for the moment the extrapolated lines, indicated by *, and weighting No. 9 with three times the possible error of the others, the mean value of $\Delta_2 = 10998{\cdot}198 - {\cdot}37\xi$, the same as from No. 10 alone which is exact. They all, with the exception of No. 5, satisfy this within observation errors $< d\lambda = {\cdot}04$. No. 5 requires that the observation error shall be ·06A and the true wave-number 19942·47 in place of ... 2·53. With this the ν_1 separation = 1775·69 and is brought into practically the exact $(\delta)\ \nu_1$ value (see p. 396) required, which is 1775·76. The outstanding ·07 ($d\lambda$ = ·017) would be attached to the strong second line of the doublet (10) 21717. This is in very striking support of the general argument. We have already seen good grounds for putting ξ a small fraction of the order ·25. To determine it with greater exactness a corresponding mantissa differing from the above by considerable multiples is necessary: *e.g.* with mantissa of order ·5 the coefficient of ξ is $15{\cdot}4\xi$. The differences equated to Δ_2 multiples would then give an equation to find ξ in which the error term would have little effect. We get this different ξ coefficient in No. 9, but it is due to an order 2 in which the effect of an error is multiplied to the same extent. The extrapolated lines do not help us as their limits of error are too large. On the contrary the argument enables us to determine their values more correctly: *e.g.* in No. (2) the error is dependent as the line 21400 from which the line is extrapolated. To make the multiple correct requires $p = 2{\cdot}7$, $dn = {\cdot}42$. This reduces the observed $\nu_2 = 809{\cdot}53$ to $809{\cdot}11$. It is supposed modified by a δ_1 shift on the sequent which here produces a change of 6·3 pointing to an original $\nu_2 = 809{\cdot}11 + 6{\cdot}03 = 815{\cdot}14$, practically exact. Applying the method to Nos. 3, 9 gives

$$
\begin{aligned}
{\cdot}98\xi &= {\cdot}06 + {\cdot}18p_3 - {\cdot}06p_3 \\
\xi &= {\cdot}06 + {\cdot}18p_9 - {\cdot}06p_3 = 06 \pm {\cdot}24
\end{aligned}
$$

with

$$\Delta_2 = 10998{\cdot}14 - {\cdot}064p_9 + {\cdot}043p_3 = 10998{\cdot}14 \pm {\cdot}10$$

But the preferable choice is to use the fact that (10) is the limit to (9), the same value of ξ must enter, and the result depends only on the observation error.

The result is now

$$ {\cdot}98\xi = {\cdot}03 + {\cdot}18p_9, \qquad \xi = {\cdot}03 + {\cdot}18p_9, \qquad \Delta_2 = 10998{\cdot}187 - {\cdot}06p_9. $$

Thus with maximum error $d\lambda = {\cdot}04$ maximum uncertainty in Δ_2 is $\pm{\cdot}06$, but the line (9) is a good one for measures and the probable error will not exceed ·02. Hence as the definitive value $\Delta_2 = 10998{\cdot}18$ is probably within ·03 and certainly within ·06. Hence

$$\Delta_2 = 10998{\cdot}187 \pm {\cdot}03, \qquad \delta = 611{\cdot}0104 \pm {\cdot}0017.$$

The value of δ obtained from the ν_1 displacement $= 610{\cdot}87 + {\cdot}76d\nu_1 - {\cdot}04\xi$, to make these the same requires $d\nu_1 = {\cdot}19$, $\nu_1 = 1778{\cdot}09$. This is possible though not probable. We cannot say definitely here therefore as in Kr that the triplet modification produces a slight difference in the deduced δ. Here the difference ·14 would correspond to a difference in mass of 27 electrons.

Radium Emanation.—The emanation does not apparently produce the two spectra exhibited by Kr and X. The measurements in the spectrum are very scanty compared with those in the latter. We have early rough determinations by RAMSAY and associates.* More accurate and complete by RUTHERFORD and ROYD†, and later by WATSON.‡ In order to diminish the absorption by the electrodes RAMSAY also used copper instead of Pt electrodes and found a number of new lines, the majority of which have not been seen by succeeding observers. They have generally been explained as due to contamination by xenon as they lie close to X lines within their errors of observation. At first sight this explanation would seem to be very natural, but RAMSAY was confident that there was no such contamination. I am inclined to suspect that the opinion that these lines belong to X is too hasty. As is well known BALY found quite a large number of lines in Kr and X coincident within his observation errors, which indeed were much smaller than those in any measures yet made in RaEm. Now as a fact those suspected lines of RAMSAY and CAMERON'S are also very close to these Kr lines. A strong argument also is this. There are a number of strong lines undoubtedly belonging to the RaEm spectrum, and observed by both RUTHERFORD and ROYD and by WATSON, which are also near strong X lines, yet separated so far from them, that if RAMSAY and CAMERON had had X in their tube they must have seen them and RaEm lines as double, one due to X and the other to RaEm. Compare for instance the following lines:—

RaEm.			X.	Kr.
C. and R.	R. and R.	W.		
(5) 4681	(10) 4680·92	(9) 4681·01	(5) 4683·76	(4) 4680·57
(10) 4626·5	(8) 4625·58	(10) 4625·66	(15) 4624·46	
(10) 4605	(4) 4604·46	(8) 4604·58	(10) 4603·21	
(3) 4578·5	(7) 4577·77	(8) 4578·0	(6) 4577·36	(6) 4577·40
(8) 4463·5	(7) 4459·3	(10) 4460·0	(20) 4462·38	(1) 4463·88
(3) 4189	(4) 4187·97	(5) 4188·2	(10) 4193·25	
(6) 4114	(6) 4114·62	(6) 4114·71	(7) 4116·25	(1) 4113·90

I therefore included these RAMSAY and CAMERON lines in the purview, with the result that a considerable number were found to fall in with places in which they are

* RAMSAY and SODDY, 'Roy. Soc. Proc.,' vol. 73, p. 346 (1904); RAMSAY and COLLIE, *ibid.*, vol. 73, p. 470; CAMERON and RAMSAY, *ibid.*, A, vol. 81, p. 210 (1908).

† RUTHERFORD and ROYDS, 'Phil. Mag.' (6), vol. 16, p. 313 (1908): ROYDS, *ibid.*, vol. 17, p. 202 (1909).

‡ H. E. WATSON, 'Roy. Soc. Proc.,' A, vol. 83, p. 50 (1909).

required by known spectrum laws. It must be remembered that the appearance of the Em lines varies very much in relative intensity with different observers (*cf.*, for instance 4604, 4460 above) that some appear early and then disappear, that others come in after the emanation has stood for a few days, and further that the copper electrodes, which extended the useful duration of the tubes, would probably have some effect on the nature of the emitting sources in the gas. To account for this, the suggestion might be thrown out that the activity of the emanation would by itself ionize the molecules of the gas, and that especially the α-rays would ionize in a different and more drastic way than the ordinary cathode or vacuum tube ionization. That with time the γ-rays from the active deposit might ionize in again a different way and produce again new lines. One would expect that the self-effect—as it may be called—is so drastic that it destroys those configurations which should give the red spectrum analogous to that in the other gases. It is a fact, as I hope to show, that the spectrum, so much as there is of it, is decidedly of the jar, or blue kind.

The degree of accuracy of the observations is not of the best. ROYD claims an accuracy of 0·1A. The spectrum was obtained by a concave grating of 1 metre radius and extended from 5084 to 3005, with some additional lines by a prism spectrograph, subject to errors of ·5A. WATSON'S lines extended from 7057 to 3867 with several new lines. His degree of accuracy is probably about the same as that of ROYD. In the following we shall treat the maximum errors as ·2A except where lines are only given to the nearest unit.

The extent of the spectrum observed is too restricted to expect to find more than the S (2) and D (1) lines, and even in the case of S (2) the S_2 (2) and S_3 (2) may be in the violet where only glass apparatus was used. Further, there is the added disadvantage that the links are so large that they can stretch from the unobserved ultra-red to the unobserved ultra-violet, and consequently can only act as sounders for lines so far in the ultra-violet that an e link lands within the visible red. The F lines should be expected to lie wholly in the observed region, and this must be the chief guide in the unravelling of the series relations.

As a preliminary and definite starting point, we have the value of the oun as calculated from the atomic weight. But here also there is some uncertainty. The value of HÖNIGSCHMIDTS' determination of the atomic weight of Ra, 225·97 is now generally accepted as close to the real value, as against the earlier value of 226·4. This makes the atomic weight of the emanation to be 222 to 222·4. These two give values of $\delta = 361{\cdot}80w^2$ as between 1783·1 and 1789·5, with the probability that it is close to 1783. The uncertainty in the value of the constant 361·80 will not affect this.

An examination of the spectrum for constant separations shows a large number of triplets with ν_1 in the region 5371 to 5383 and ν_2 at 2671 and less. Further, the higher values appear in sets which show inverted order of intensities. This suggests that the lines belong to D satellite systems and that the separations about 5383, 2671 belong to the modified ν_1, ν_2 which D satellites have already shown in Kr and X, whilst 5371, 2641 or thereabouts belong to the normal separations depending on displacements in the S (∞) alone. Moreover, another very frequent separation is 5631, connected with other sets as triplets with a ν_2 in the neighbourhood of 2800. This at once suggests the analogue of the 1864 F series of X.

A first quite definite starting point, from the material at disposal, is found by a search for lines of the F and **F** type, or the twin A±B sets. The limit A belongs to a D sequence, which from analogy with Kr and X should be expected to be of the order $n = 30000$. Now in the observed spectrum there is a long gap between 27671 and 32031, within which such limit must be. That no lines should be found near this limit is to be expected. If, however, such double sets exist we should expect to find sets of lines with exactly the same separations on either side of this gap. Unfortunately there are only four lines on the violet side, but one such set is found. They are

(1) 26669·23		(0) 33259·05	
	897·13		**896·91**
(2) 27566·36		(0) 32362·14	

The corresponding limit should be the mean of either of the two corresponding lines, viz., 29964·14 or ...4·25, say 29964·20 ± ·05. The possible observed errors in these lines are not large and the practically exact equality of the two separations is strong evidence of the reality of the suspected connection. But any doubt on this point must be removed when it is noted that by RYDBERG'S tables, the separation 897 is that due to two denominators 5·78, 6·78, whilst if the denominators are calculated using 29964 as limit the same values are found. The two results are quite independent. By a further use of RYDBERG'S tables it is possible to find approximate positions for other lines of the F system, the **F** being quite beyond the observed region in the ultra-violet. Such lines are found at (5) 15846, (4) 22310·4, (8) 25171 for $m = 2, 3, 4$. Again, connected with 15846, are (3) 21488, (6) 24296·9 giving separations 5642, 2809. The lines 22310, 21487 are due to C. R.'s copper electrodes and are subject to considerable possible errors $d\lambda = 1$A, $dn = 5$, or even more. These strikingly correspond to the 1864 sets in XF.

The limit 29964·20 must be very accurate and subject only to any systematic errors in R. and R.'s measurements. This is shown by the exactness of the observed separations in sets so far removed from one another as 26669 and 32362. Using this limit with the lines 15846, 26669·23 for $m = 2$ and 5 the calculated formula is

$$n = 29964{\cdot}20 - \mathrm{N} \Big/ \left\{ m + {\cdot}757457 + \frac{{\cdot}059446}{m} \right\}^2 .$$

The two lines used may be regarded as having possible errors $dn = 2$ and ·7 respectively and any consequent errors in the constants will scarcely affect the calculated frequencies for $m > 2$. These for $m = 3, 4, 6$ are 22277·28, 25148·60, 27569·40. The last gives $d\lambda = {\cdot}40$. No lines correspond to the others, but 22211 observed by C. and R. with Cu electrodes and (15) 25107·14 (R.), (9)...6·64 (W.) are respectively 66·28 and 41·56, (41·96) less. Now a δ_1 displacement on the limit gives 13·96, so that these correspond within error limits to a $(5\delta_1)\,F(\infty)$, $d\lambda = -{\cdot}5$, and $(3\delta_1)\,F(\infty)$, $d\lambda = -{\cdot}06$, and stand in some analogy to what has been seen to happen in X. The intensity of the second line however would seem very great for $m = 4$. In support of the C.R. line with its large possible error is (2) 22196·92 (ROYD) at 13·08 behind it or an extra δ_1. It would make $22196 = (6\delta_1)\,F$, and consequently $F = 22280{\cdot}68$, *i.e.*, $d\lambda = -{\cdot}7$. If the 5640 lines exist, they all lie outside the region of observation except in the case of the first line 15846 where the corresponding triplet set is found (see above). However, anticipating the value of the links obtained later and using them as sounders, viz., $e = 23678{\cdot}3$, $u = 11191{\cdot}8$, $v = 13680{\cdot}6$ with some uncertainties we can test for their existence. Taking the observed lines 22196·92, 22211 we expect lines about 27838, 27858, only to be observed if strong. There are none, but there are (3) 14166 (W.) **2816** 16982·0 (C.R.) which with the v link give $27846{\cdot}6 \pm 1{\cdot}9 + dv$, $30662{\cdot}6 \pm 3 + dv$, separation $= 2816 \pm 5$. These suggest the triplet set $(\delta_1)\,F$, viz.,

22196·9	**5649·7**			
		$27846{\cdot}6 + 1{\cdot}9p + dv$	**2816 + 3p′ − 1·9p**	$30662{\cdot}6 + 3p' + dv$
22210·9	**5635 + 1·9p + du**			

The results of sounding are indicated in the following, where, since e, u, v are only approximately known, their values are supposed corrected by de, du, dv. The letters after wave-lengths refer to observers :—

$$m = 3. \quad (5\delta_1)\,F.$$

Outside		(3) 14166 (W.).v		16982 (C.R.).v
22196·9	**5649·7**			
22210·9	**5635 + 1·9p + dv**	$27846{\cdot}6 + 1{\cdot}9p + dv$	**2816 + 3p′ − 1·9p**	$30662{\cdot}6 + 3p + dv$

With the calculated value of F_1 the first separation is 3·5 larger. The first line is δ_1 displacement on 22210. If the former does not belong to the system, we may take C.R.'s direct line which has possible error of 4 or 5.

$$m = 4. \quad (3\delta_1)\,F.$$

1.	Outside		$(2\delta_1)$ 19583 (C.R.).u		$(2\delta_1)$ 22398 (C.R.).u
	25107·14	**5640·6 + 4p + du**	$(30747{\cdot}8) + 4p + du$	**2814·2 + 5p′ − 4p**	$(33562) + 5p + du$
2.	v.(0) 15136 (W.).e		v.(0) 20784·31 (W.).e		v.(0) 23598·2 (W.).e
			$v.(-\delta_1)$ (1) 20799 (C.R.).e		
	(25134·3)	**5648·31 + de − dv**	(30782·6)	**2814·9**	(33596·5)
3.	[25148·60]	**5648**	v.(1) 20799.e	**2816·0**	u.(1) 21126·0.e

No. (2) is close to $(-2\delta_1)$ 25107 or (δ_1) F_1. It should be noted that the C.R. copper line is a $(-\delta_1)$ displacement on that observed by WATSON, and that the latter was only observed after the emanation had been standing two days. No. 3 gives the calculated values.

$m = 5.$

1.	Outside (1) 26669·23	**5648·6+du**	(1) 21126·0 (W.).u 32317·8	**2822+6p**	23948 (C.R.).u 35139+6p	
2.	26669·23	**5652·8+dv**	(2) 18641·4 (W.).v 32322·0	**2817·4**	(6) 21458·86 (W.).v, (6·56) (R.R.) 35139·46	
3.	26669·23	**5659·7+de−du**	u.(1) 19842·4 (W.).e 32328·9	**2793·6+6p**	u.(4) 22636 (C.R.).e 35122·5	

These would seem to show in Nos. 2, 3 sequence displacements; 2δ would here produce a change of 4. In (2) it is displaced in the second and continued in the third. In (3) it is displaced only in the second. 21126 acts as a sounded line for $F_3(4)$ and $F_2(5)$. Both cannot be real or the line is double.

$m = 6.$

1.	(2) 27566·36	**5640·4+6p+du**	(1) 22015 (C.R.).u 33206·8+6p
2.	27566·36	**5642+dv**	(3) 19527·8 (W.).v 33208·4

There is a C.R. copper line (2) 16386±3 which with the u link gives 27577±3 and may be $(-\delta_1)$ F (6).

The two last observed lines have been included in order to show that sounding is justified by them. The calculated line for $m = 7$ is 28155·7, with error probably within 3 or 4 units. There is 16982 (C.R.) which with u sounder gives 28173·8±3 and may well correspond to (δ_1) $F_1(7)$. Also 22636 (C.R.).u gives 33827·8 which is 5654 ahead and may be (δ_1) $F_2(7)$. But this line with an $e-u$ link also gives $F_3(6)$. It should be noticed how the lines observed by CAMERON and RAMSAY with copper electrodes come in to fill parallel and displaced lines where they seem called for.

The corresponding **F** lines for the orders above the observed ones are in the ultra-violet, but evidence for their existence is given by sounding. In what follows, the value of $\mathbf{F}_1$ calculated from F_1 is enclosed in square brackets.

$m = 2.$ [44082·4±2.]

1.	(δ_1) (6) 21712·09 (R.).2u 44081·7+2du	**5649·6**	$(3\delta_1)$ (0) 27389·61 (R.).2u 49731·3+2du		
2.	(3) 16725·1 (W.).2v 44086·3+2dv	**8445 = 5645+2814−13·96**		(8) 25170·10 (W.).2v ? 52531·3+2dv	

In No. 1 the sounded line $\mathbf{F}_2$ is on the verge of the observed region. In No. 2 the sounded is about 4 too large, and there appears a δ_1 displacement on the last. The u, v links themselves are too short to reach.

$m = 3$. [37650·6 from Calculated F, 37717 from Observed $(5\delta_1)$ F.]

(3) 23973·69.v		(0) 15944 (W.).$2v$		(2) 18739 (C.R.).$2v+4p$
37654·29 + dv	**5651 + dv**	43305·2 + $2dv$	**2795 + 4p**	46100·2 + $2dv$

Here the reproduced line refers to the normal line calculated from the formula. The v link is too short for the second and third lines, $2v$ reaches it, but $2v$ on the first would require a reference line in the ultra-red. Also 15944 is possibly $(-7\delta_1)$ F_1 (2).

$m = 4$. [34779·80.]

(3) 23584 (C.R.).u } (δ_1) (0) 23598·2 (W.).u }				$(-2\delta_1)$ (3) 16725·1 (W.).e
34776·0		**5655·2 + de − du**		40431·2

Again note a C.R. copper line supported by a W. and R.R. displaced δ_1 line

$m = 5$. [33259·17.]

		(2) 25213·7 (W.).v		(0) 18024 (W.).e
33259·05	**5635·25 + dv**	38894·3	**2808·0 + de − dv**	41702·3

Here appears the frequent 5635. It is 5649 − 14, that is, there is a limit displacement of δ_1 in the second line and an extra one in the third, making the second separation 2816.

$m = 6$.

The same links cannot serve as sounders for all three lines.

For $\mathbf{F}_1$ is (2) 211725·4 (W.).u = 32364·34 + du as against calculated 32362·14.
For $\mathbf{F}_2$ is v.(2) 16816·5 (W.).$e.u$ = 38004·8 + $de + du - dv$.
For $\mathbf{F}_3$ (4) 18448 (C.R.).$2u$ = 40831·6 + $2du$.

These give separations 5640·5, 2826·8, where the sum has the normal value. In $(-\delta_1)$ (3) 23973·69 (W.).u = 35178·45 which is 2816·2 ahead of $\mathbf{F}_1$ we have the completion of a mesh with the other three lines.

The foregoing discussion has shown: (1) that this F set belongs to the 1864 type discovered in X; (2) that the usual displacements are present and that the calculated value per oun—13·96—satisfies all the numerical relations formed; and (3) that the lines observed with copper electrodes by CAMERON and RAMSAY seem to belong specially to parallel series to this set. We could feel complete confidence in the allocation of the lines were it not that the α constant in the formula is positive, and that the line $m = 6$ is not reproduced more closely. The absence of direct representatives for $m = 3$, 4 is not surprising as their limit displaced values are certainly observed and the change is in full agreement with what takes place in the other elements.

We shall assume in what follows that the preceding allocation is correct, in other words the limit is $29964{\cdot}20+\xi$, the line for $m = 2$ is $15846+2{\cdot}5p$, and that the series belongs to the F type. In that case the mantissa of the limit and of the sequent are both multiples of the oun. These mantissæ are respectively $913165-31{\cdot}92\xi$ and $787174+246p-98{\cdot}7\xi$. As both are oun multiples, so must be their difference. This difference is

$$\begin{aligned}125991-246p+68\xi &= 70\tfrac{1}{2}\,(1787{\cdot}10-3{\cdot}49p+{\cdot}97\xi)\\ &= 70\tfrac{3}{4}\,(1780{\cdot}75-3{\cdot}49p+{\cdot}97\xi)\end{aligned} \quad (1)$$

In which if WATSON is correct to nearest unit p is equally probable between $\pm{\cdot}5$.

It is very unfortunate that here we have to deal with two uncertainties not generally met with, viz., on the one hand the uncertainty as to the real value of the atomic weight, and on the other the magnitude of the possible observation error in the fundamental wave-length, which WATSON has only measured to the nearest Ångström. If this had been ·1, *i.e.*, $p = {\cdot}1$, the above result would show that since δ lies between 1789 and 1783, the multiple must be $70\frac{1}{2}$ without any doubt, and consequently δ in the neighbourhood of 1787. The value of ξ is so small, that its term will not affect our present reasoning. We have, however, to allow for this uncertainty and a value of $p = -{\cdot}7$ makes the second multiple $= 70\frac{3}{4}\times 1783{\cdot}2$ with a possible δ. In this case the first multiple gives $70\frac{1}{2}\times 1789{\cdot}55$, or δ just on the improbable limit and it might be excluded. The result, therefore, is

Equally possible, $p<{\cdot}5>-{\cdot}5$, multiple $= 70\frac{1}{2}$ and δ between 1785·3 and 1788.
Improbable, but perhaps possible, $p = -{\cdot}7$, multiple may be $70\frac{3}{4}$ and $\delta = 1783{\cdot}1$.
Very improbable, $p = 1$, multiple $70\frac{1}{2}$ and $\delta = 1783{\cdot}6$.

But also the limit and sequent mantissæ must also be oun multiples, now

$$\begin{aligned}913165-31{\cdot}92\xi &= 512\,(1783{\cdot}52-{\cdot}062\xi)\\ &= \ldots\\ &= 511\,(1787{\cdot}016-{\cdot}062\xi)\end{aligned} \quad (2)$$

$$\begin{aligned}787174+246p-98{\cdot}7\xi &= 441\tfrac{1}{2}\,(1782{\cdot}95+{\cdot}55p-{\cdot}22\xi)\\ &= \ldots\\ &= 440\tfrac{1}{2}\,(1787{\cdot}00+{\cdot}55p-{\cdot}22\xi)\end{aligned} \quad (3)$$

It might occur to the reader that the last should be a multiple of Δ_2. But if the series is the analogue of the 1864XF, to which the foregoing argument has pointed, it should have a line of order $m = 1$ (n about $= 3260$). This should show $M(\Delta_2)$.

The limit condition is independent of p and can only be modified by ξ in the second decimal place. It gives quite definitely six possible values for δ, viz., 1783·52, 1784·39, 1785·26, 1786·13, 1787·01, 1787·87 with multiples 512 diminishing by $\frac{1}{4}$ to $510\frac{3}{4}$. Of these the following can be satisfied by the mantissa difference condition (1)

1783·5 by $p = -·8$, multiple $70\frac{3}{4}$, not probable.
1783·5 by $p = 1$, multiple $70\frac{1}{2}$, very improbable.
The last four by $p > -·25 < ·5$, multiple $70\frac{1}{2}$, equally probable.
The others by $p > ·5 < 1$, multiple $70\frac{1}{2}$, improbable.

If WATSON'S readings are really to the nearest unit, $p = \pm ·5$. This probable consideration would largely reduce the limits of uncertainty. It would in conditions (1) exclude the second and with multiple $70\frac{1}{2}$ give 1787·0 with $p = 0$, 1786·1 with $p = ·3$, 1785·3 with $p = ·5$. Conditions (3) would then of these give 1787 with $p = 0$, multiple $440\frac{1}{2}$, 1786·16 with $440\frac{3}{4}$, 1785·27 with 441. All of these have equal probability, but they exclude the 1783 based on HÖNIGSCHMIDTS' atomic weight. The lowest value 1785·3 would make the atomic weight = 222·15 ± ·02, and that of Ra = 226·15 as compared with HÖNIGSCHMIDTS' 225·97.

Before passing from this series it will be important to get as close an estimate as possible of the two separations. Regarded as oun displacements on the limits they should give some further data for the determination of the oun—or, vice versa. The separations given in the sounding operations above are here collected. Errors from WATSON'S, or BALY'S observations will not amount to more than a few decimals at the outside.

F.			F.		
1.	5649·7 } $+1·9p+dv$ 35·7 }	$2816+3p'-1·9p$	9.	5649·6	
2.	$40·6+4p+du$	$14·2+5p'-4p$	10.	$50·9+dv$	$2795+4p$
3.	$48·31+de-dv$	14·9 $16·0-du+dv$	11.	$55·2+de-du$	
4.	$48·6+du$	$22+6p$	12.	$35·25+dv$	$2802·1+de-dv$
5.	$52·8+dv$	17·4	13.	36·6	33·0
6.	$59·7+de-du$	$793·6+6p$			
7.	$40·4+6p+dv$				
8.	$42·0+dv$				

The ν_1 cluster round 5649, 40, 35 and the ν_2 around 2816.

In ν_1 the 49 and 35 differ by 14 and are clearly due to a δ_1 displacement in the limit. We will consider the exceptions in order. In (2) there is an uncertainty $4p$. If $p = -1$ we get, with other errors close to 35, in this case $\nu_2 = 18·2+5p'$ and a small error in p' brings it to the 16 neighbourhood. In (4) the uncertainty $6p$ reduces again ν_2 to the 16. In (5) ordinary errors bring ν_1 to the 49 value. In (6) modification of 10·7 in the middle line brings ν_1 to 49 and $p = ·25$ brings ν_2 to 16 or $\nu_1+\nu_2$ = normal. In (7) the error allows 35. No. 8 does not seem amenable and may not therefore be a real connection. In (10) ν_1 is a normal 49 with error 2 and ν_2 with $p = 1$ is $00 = 14-14$ or the oun displacement in the last line. In (12) also the oun displacement in the last line makes $\nu_2 = 16$. No. 13 can be explained by the oun displacement on the middle line alone. They can all then with the exception of one be explained by $\nu_1 = 5635$ or 5649, and $\nu_2 = 2816$ within limits of about ± 1. The consideration of their source is postponed until the D sequents have been touched upon (p. 441).

An examination of the spectral list shows values of ν_1 varying between 5361 and 5388 with ν_2 from 2639 to 2685. In many cases two lines are separated by about 5371 and a line approximately midway with separations $2680\pm$. Now in the other gases the appearance of such $\frac{1}{2}\nu_1$ separations is so common as to be almost the rule. This lends weight on the one side to the assumption that $5371\pm$ is ν_1 and on the other that 2680 is not ν_2 but corresponds to $\frac{1}{2}\nu_1$. Values therefore in the neighbourhood of 2686 may be left out of account in the search for ν_2 which is always less than $\frac{1}{2}\nu_1$.

There are a considerable number of separations in the neighbourhood of $5380\pm$ which again suggest the analogous modified ν_1 related to the D satellites in Kr and X. In those cases the explanation was adopted that at least in the main they arose from displacements of small multiples of the oun in the sequence term. No exact value of these displacements can be obtained until the value of the limit itself is known, but it is possible to arrive at an approximate estimate by employment of a value of the limit which may be a few hundreds wrong. Such an estimate will be of great value in guiding our search.

The values of $S(\infty)$ for A, Kr, X are respectively 51731, 51651, 51025. From analogy, that for Ra, Em would be in the neighbourhood of 50500, say $50500+\xi$. With ν_1 in the neighbourhood of 5371 ± 1 calculation shows that a displacement of δ_1 will produce a change in the limit of $30{\cdot}86+{\cdot}0011\xi$ and in the ν_1 of $4{\cdot}72$. The D sequence terms vary with each satellite set. One such sequence set has already been found in the F series already discussed, viz., 29964. Here the oun displacements produces a change in n of $13{\cdot}96$. But higher values than this for $d(1)$ sequents are to be allowed for up to even 31000. For 30500 the oun displacement produces a change of $14{\cdot}88$. It is safe therefore to take the oun displacement on the sequent as producing a change varying from 14 to 15.

If now the $5370\pm$ separations be analysed it will be found that three lie at $71{\cdot}4\pm{\cdot}4$, eight at $74{\cdot}2\pm{\cdot}6$, five at $80\pm{\cdot}5$, six at $84{\cdot}2\pm1$ and a group at 88 with none between. There is a clear majority about 74 with a set at 88, or about 14 ahead (*i.e.*, $-\delta_1$ on the sequent) and a few about 61, or 14 behind. Whilst the $71{\cdot}7$ and 78 sets suggest the δ_1 displacements in the limits, definitely seen to exist in Kr and X.

For the ν_2, the oun changes on the sequent are the same and on the limit are about half the previous, say, about 2 or 3. In this case are found a majority near 2649 with some about 14 on either side, and another few about 2652.

The material at disposal then goes to show that the S separations are near 5371, 2649 for some definite limit and 5374, 2652 for another whose limit mantissa is one oun less.

Arguing from analogy with the successive spectra of A, Kr, X we should expect to find in the observed region only lines corresponding to D (1) and S (2). The D (2) lines would be considerably shorter than the last observed line $n = 33259$. The S (1) would be in reversed order with $S_1(1)$ near -42100 or $S_3(1)$ near -34000, the first absorbed by the glass apparatus used. We should expect D (1) lines up to the longest observed ($n = 14166$) with $D_{11}(1)$ lines down to at least $n = 21000$, and showing the same kind of modified separations as in previous cases, and taken account of above. $S_1(2)$ should be about 25600 with $S_2(2)$ about 31270, just on the observed boundary and $S_3(2)$ quite beyond about 33900. This absence of S separations is the reason why it was so difficult above to obtain accurate values of ν_1, ν_2 for a definite limit. By themselves therefore the material is hopelessly inadequate to determine the $S(\infty)$ limit, the values of Δ_1, Δ_2, or of the various links. We have only five possible—or three probable—choices for δ to 5 significant figures, and also the value of one $d(1)$ sequent correct to a few decimals, with estimates of the F and S triplet separations. The only method of attack then seems to be an indirect one, to tabulate the sets of lines giving the triplet separations, to try to distinguish between those related to D and S systems, to obtain as close a value as possible of the ν_1, ν_2, to determine some of the satellite separations, and from these last to attempt to find the corresponding F series with the same constant separations. These F series ought in each set to consist of several orders (m) at least, as the $F(\infty)$ all lie in the observed although badly observed region. The observed separations and the values of ν_1, ν_2 combined with the approximate value of the oun may enable a determination of the important constants $S(\infty)$, Δ_1, Δ_2 to be arrived at.

We shall take then $5371+d\nu_1$ $2649+d\nu_2$ and a set about 3 larger for the values of ν_1, ν_2, the two sets belonging to two limits, relatively displaced by an oun, and both giving the same values of Δ_1, Δ_2. In the above the $d\nu_1$, $d\nu_2$ will probably not be greater numerically than 1. The calculations will be made with the 5371 set and the conditions applied $d\nu_1 = \pm 1$, &c., and $d\nu_1 = x \pm 1$, $d\nu_2 = y \pm 1$ where x, y are the changes produced by the oun displacement. We start the first approximation by taking $S(\infty) = 50500+\xi$ where ξ may amount to several hundreds and in which x, y are of the order 4·7, 2·2. The denominators of the S_1, S_2, S_3 limits with their differences calculated from these are

$$1{\cdot}473697 - 14{\cdot}591\xi$$

$$72624 - 2{\cdot}053\xi + 12{\cdot}54\, d\nu_1$$

$$1{\cdot}401073 - 12{\cdot}538\,(\xi + d\nu_1)$$

$$32076 - {\cdot}839\,(\xi + d\nu_1) + 11{\cdot}70\, d\nu_2$$

$$1{\cdot}368995 - 11{\cdot}699\,(\xi + d\nu_1 + d\nu_2)$$

These differences may be written

$$72624 = 40\tfrac{3}{4}(1782{\cdot}18 - 050\xi + {\cdot}31 d\nu_1) \qquad 32076 = 18\ (1782{\cdot}00 - {\cdot}047\xi - {\cdot}047 d\nu_1 + {\cdot}65 d\nu_2)$$
$$= 40\tfrac{1}{2}(1793{\cdot}18 \ldots \quad ,, \quad) \qquad = 17\tfrac{3}{4}(1807{\cdot}1 \ldots \quad ,, \quad)$$

The first consequence to be drawn from these numbers is that as δ is to be of the same order of magnitude as calculated from Δ_1 and Δ_2 it can only take place in the first arrangement since the co-efficients of ξ (the only variable capable of large values) are practically equal. In other words $\Delta_1 = 40\tfrac{3}{4}\delta$, $\Delta_2 = 18\delta$, the same multiples as in X. This is one quite definite conclusion of importance.

But further we know that δ must be one of the following sets 1783·5, 84·4, 85·3, 86·1, 87·0, of which the first two though perhaps possible are improbable and the last three equally probable. We will state the results for the first, third, and fifth, the others being intermediate. The values of $d\nu_1$, $d\nu_2$ being uncertain to about unity, the values of ξ can only be estimated to the same order. Taking Δ_1, in order to produce the required values of δ, ξ must have the respective values of -27, -64, -100 within a few units, and the numbers inside the brackets for Δ_1, Δ_2, become

$$1783{\cdot}53 - {\cdot}050\xi + {\cdot}31 d\nu_1 \qquad 1783{\cdot}27 - 047\xi - {\cdot}047 d\nu_1 + {\cdot}65 d\nu_2$$
$$1785{\cdot}38 - \ldots \quad ,, \qquad 1785{\cdot}00 \ldots \quad ,,$$
$$1787{\cdot}18 - \ldots \quad ,, \qquad 1786{\cdot}7 \ldots \quad ,,$$

where now ξ cannot exceed a few units. The respective limits become 50473, 50436, 50400. One of these is close to a S limit. There is another corresponding to the $-\delta_1$ displacement about 31 ahead and giving $\nu_1 = 5375$ about. We already have an accurate value of one d sequence, viz., the F (∞) = 29964·20. Combined with the limit this should give a D line = S (∞) $-$29964·20, *i.e.*, near 20509, 20472, or 20435, or the 31 higher values. We only find the following possible lines, viz. (2) 20515 and (4) 20438·8 (W.), (0) 20446·38 (R.R.), 20473, the first and fourth being C.R.'s copper electrode lines. The first is one of a chain of 2680 separations, or $\frac{1}{2}\nu_1$ stretching back to 15136 and by 2689 to the strong line (10) 23204·6 (R.) or 7·57 (W). It has none of the signs of a D line. The line 20438 is one of the strongest in this neighbourhood has no ν_1 separation link, but 5625·8 back to 14813 $\pm$ 2 (W) to which also 20446 has the (δ_1) 5649 link, viz.,5633. Again the 14813 shows a D modified ν_1, ν_2 set, viz.,

(0) 14813 (W)	**5382·71**	(4) 20195·71 (W)	**2672·41**	(5) 22868·12 (W),
	5385·91	(0) 20198·91 (R)	**2669·16**	(1) ·07 (R).

Hence here we have all the signs of a D_{11} line in 20438 with the above as a satellite set, at the F separation (56...) behind. Now it is a very striking fact that the measures of W. and of R.R. for these lines not only differ by amounts larger than can be attributed to observation errors, but also that their intensities are not comparable. Where W. gives 20438·8, with intensity 4, R.R. give 20446·38 with intensity (0), and a similar effect is shown in the second line of the above triplet (R.R. have not observed so far in the red as 14813). The differences 5·6, 3·2 are comparable with the differences shown by the ν_1 sets roughly estimated above, say 5371, 5374, and would seem to be due to the same effect, although this would not go with the explanation there suggested. Further R.R.'s 20446·38 is 5633·4±2 ahead of 14813 and is therefore one exact determined value of the F_2 separation. We are justified therefore in taking 20438 as a D_{11} line belonging to the $S(\infty) = 50400+\xi$. With $d = 29964{\cdot}20+\xi'$ the calculated D line is therefore $20435{\cdot}80+\xi-\xi' = 20438{\cdot}8+{\cdot}4p$

$$\xi = 3{\cdot}0+{\cdot}4p+\xi'$$

where ξ' is small. Hence $S(\infty) = 50403{\cdot}00+\xi$ where ξ is small and equal to $\xi' \pm {\cdot}4$. A reliable value of this limit has thus been obtained. The measure of its reliability is that of two assumptions (1) that the 5649 (or 34) series studied at the beginning of the discussion of this element is an F type analogous to the 1864 of X, and (2) that 20438* is a D_{11} line. With this, $S(\infty) = 50403{\cdot}00+\xi$ and $\nu_1 = 5371+d\nu_1$, $\nu_2 = 2649 + d\nu_2$ the denominators and values of Δ_1, Δ_2, calculated directly are 1·475114, 1·402290, 1·370130 and

$$\Delta_1 = 72824-2{\cdot}06\xi+12{\cdot}571d\nu_1 \qquad = 40\tfrac{3}{4}\{1787{\cdot}09-{\cdot}050\xi+{\cdot}31d\nu_1\}$$
$$\Delta_2 = 32160-\ {\cdot}84(\xi+d\nu_1)+11{\cdot}726d\nu_2 = 18\ \{1786{\cdot}66-{\cdot}046(\xi+d\nu_1)+{\cdot}651d\nu_2\}.$$

The δ as determined from the Δ_2 has always been the same as those found from the D and F mantissæ, here $1787{\cdot}015-{\cdot}062(\xi-{\cdot}4p)$. Hence $d\nu_2 = {\cdot}53$ correct to the first decimal place and $\nu_2 = 2649{\cdot}53+d\nu_2$ where $d\nu_2 < {\cdot}1$ and $\Delta_2 = 32166$. If the δ from Δ_1 is the same $d\nu_1 = -{\cdot}3$ and $\nu_1 = 5370{\cdot}7$, $\Delta_1 = 72820$. Wherever it is different it has always been slightly less than that from Δ_2, so that we may regard 5370·7 as a maximum estimate for ν_1. It may be a few decimals smaller, but we have no direct means from observed lines to determine it more closely.

A similar treatment with the four equally probable values of δ are appended. Also changes in $d\nu_1$, if δ from ν_1 is the same as from ν_2. This will at least give maximum values of ν_1. The cases are given for the limit used above and also in the last two columns for that if 20446 is the D_{11} line, that is the limit about 7 larger.

	$d\nu_1$	$d\nu_2$	$d\nu_1$	$d\nu_2$
$1787{\cdot}87-{\cdot}062\xi$	2·5	1·7	3·5	2·5
1787·01–	– ·3	·5	·9	1
1786·13–	–3	– ·8	–2	– ·3
1785·26	–6	–2·1	–5	–2

* Possibly not so certain. 20446 gives one of the normal 5633 to 14818. Its weak intensity may be due, as in the other gases, to the presence of displacements and so may be D_{11}. It would make $S(\infty) = 50410$.

The limit 50403 and $\delta = 1787{\cdot}01$ require the smallest changes and seems preferable, although limit 50410 and $\delta = 1786$ is perhaps possible. Both 1787·87 and 1785·26 would appear excluded. The numerical work to follow will be based on the limit 50403, $\nu_1 = 5370{\cdot}7 + d\nu_1$, $\nu_2 = 2649{\cdot}5 + d\nu_2$, $S(\infty) = 50403{\cdot}00 + \xi$. These make $\Delta_1 = 72820$, $\Delta_2 = 32166$.

With these more exact values of $S(\infty)$ and ν_1 it is now possible to obtain the important constants, the *p*-links. They are found to be

$$a = 4630{\cdot}72 - 4{\cdot}11x + {\cdot}010\xi + {\cdot}74 d\nu_1 \qquad c = 6277{\cdot}21 - 6{\cdot}16x - {\cdot}020\xi + 1{\cdot}347 d\nu_1$$

$$b = 5370{\cdot}70 - 5{\cdot}01x \qquad d = 7399{\cdot}60 - 7{\cdot}67x - {\cdot}046\xi + 1{\cdot}822 d\nu_1$$

$$e = 23678{\cdot}42 - 22{\cdot}96x - {\cdot}056\xi + 4{\cdot}912 d\nu_1$$

where the terms in x denotes the changes for a displacement of $x\delta_1$ in $S(\infty)$.

To determine the u, v links requires a knowledge of $S_1(1)$. It is impossible to obtain really definite information on this point from direct observations. In default of this the following considerations will give some indications, and will serve to illustrate how the laws of relationship already determined can give clues indirectly. In the cases of A, Kr, X the lines for $S_1(1)$ are respectively -42642, -42469, -42153. The corresponding line for RaEm should therefore be looked for about -41800, with $s(1) = P(\infty)$ about $50403 + 41800$, or, say 92200. The *e* link unfortunately is too large to test for the lines for it would reach back to lines about 11103 **2649·5** 12751·5 **5370·7** 18122·2, of which the last only would be in the observed region. Consequently no evidence can be obtained with reference to it from the ν_1, ν_2 separations. There is a strong line at (8) 17909·9. It would give $S_1(1) = 41586$, smaller than seems likely. Also it has a 5637 link not usually connected, so far as we yet know, with S lines. It however is the only strong line in the neighbourhood, and it will be probably wiser to conclude that the S (1) line has no linked *e* line to it. The above value for $P(\infty)$ gives a v link about 13660, and this link on the supposed S (1) should produce a line about 28140. We find the set

(2) 20163·7 **2645** (4) 22808·68 [28183·7]

in which the last is extrapolated by the normal $\nu_1 + \nu_2$ from the first.

It falls in the large gap where no lines have been observed and the spectroscopic apparatus was defective. If this be really the $S(1) - v$ set, it is possible to calculate what the u, v links are. Thus $S_1(1) = -(28183{\cdot}7 + v)$, $s(1) = P(\infty) = 78586{\cdot}7 + v$, and v is produced by the $-\Delta_1$ displacement in the denominator of this. As v by a rough determination is in the neighbourhood of 13660, it can be put $= 13660 + x$ where x is not very large and can be determined by the condition that $13660 + x$ is the change produced in $92246+x$ by a decrease of $\Delta_1 = 72820$ in its denominator. The result is that

$$P(\infty) = S(1) = 92266 = N/\{1{\cdot}090264\}^2$$
$$u = 11191{\cdot}8$$
$$v = 13680{\cdot}0$$
$$S_1(1) = -41864{\cdot}3$$

The set of S (1) lines would then be

−41864·3, −36493·6 or 89·7,* −33844·1.

On the two latter u, $e-u$, $e-v$ links as sounders should give the following lines, below which are placed the nearest observed,

$-u$	25301·8 or 297·3	22652·3
	(2) 25299·1±·7 (W.)	(4) 22636±6 (C.R.)
$-(e-u)$	24007·1 or 3·2	21357·6
	(20) 23993·82 (R.) 4·45 (W.)	(10) 21357·43 (R.) 6·57 (W.)
$-(e-v)$	26495·3	23846
	...	(2) 23841·9 (W.)

They are not satisfactory sounded lines, but are given as material at disposal only, $u.S_2$ and $e.S_3.u$ are really good.

There is a striking triplet which has all the aspect of being a S (1) set except that of position. It is

(2) 18611·7 (W.)	**2648·9**	(5) 21260·21 (W.)		
(0) ... 09·94 (R.)	**2652·3**	(3) 21262·25 (R.)	**5371·3**	(10) 26633·64 (R.)

Its separations give support to the values obtained above. It is a parallel set to our adopted S (1), separated from it by 15230·7. I have not been able to recognise any arrangement of links which give this value although $2b+a$, and $e-v+b$ are close to it.†

* Sounded from observed line.

† There is the possibility to be kept in mind that it may be a real and independent S set of lines, not analogous to the set considered in the previous elements. If so, it must have the same limit and $s(1) = 77036{\cdot}64 = N/\{1{\cdot}193177\}^2$. Now the s sequence depends on the atomic volume, although the exact relation is not known. In the next group, the alkalies, the denominator is of the form $\cdot 987+kv$, where v is a number proportional to the atomic volume. If here the group constant be $\cdot 99$, the denominators of the two types of s sequences can be expressed as $\cdot 99+kv$ and $\cdot 99+2kv$. In other words, this new type may depend on twice the atomic volume.

$S_1(2)$ should be a strong line in the neighbourhood of 25900, with $S_2(2)$, $S_3(2)$ in the ultra-violet. There are three strong lines in this region, viz. (8) 25262·73, (10) 25769·9, (10) 26633·64. The last is that which has been seen to be a parallel to $S_1(1)$. The first line gives a mantissa less than that for $S_1(1)$, and therefore would make the α constant positive, in opposition to all experience for S series. We are left therefore with 25769·9±3, but this is a C.R. copper line and so far dubious. If, however, it be taken as $S_1(2)$ the formula for the S series is

$$n = 50403 - \mathrm{N} \Big/ \left\{m + \cdot 129855 - \frac{\cdot 039596}{m}\right\}^2.$$

The value of $S_1(3)$ calculated from this is 39112·08. Sounding with $-v$ gives 25432·08. WATSON gives a line at (0) 25432·3±1 and this is corroborative so far as it goes, but the formula is out of step with those for A, Kr, X, and does not give confidence. If, however, lower intensities are admitted it is possible to obtain lines which fall in excellently with all the conditions, and moreover indicate other displaced sets. We find the lines, to which extrapolated ν_1 lines are added

(3) 25416·89 (R.)	ν_1	[30788·59]	(0) 25432·35 (W.)	ν_1	[30803·05]
(2) 25425·51 (W.)	ν_1	[30796·21]	(0) 25457·0 (W.)	ν_1	[30827·7]
			(1) 25453·1 (R.)		

If the extrapolated lines are sounded for by $-v$, the 30803·05 should give a line at 17123·05. This was seen by W. at (0) 17123·0, the nearest to this being (1) 17150·93 (R.) which might possibly be within error limits of v.[30827·7]. If, with this indication of 25432·35, 30803·0 as part of a triplet set suitable for S (2) we calculate the formula with the previously alloted $S_1(1)$ and $S(\infty)$, it comes to

$$x = 50403 - \mathrm{N} \Big/ \left\{m + \cdot 101230 - \frac{\cdot 010966}{m}\right\}^2$$

which is in close analogy with that for the other elements. For $m = 3$ it gives 38972·54. A v-sounder requires a line at 25292·54. ROYDS has observed (3) 25292·12, and again in this neighbourhood we find some close lines observed by only one of W. or R., viz., (8) 25262·73 (W. and R.),* (2) 25299·1 (W.) again evidencing the presence of displacements. For $m = 3$ the oun produces a change in the sequent of 3·30, so that 25299·1 which is 6·66 above $S_1(3)$ is $S_1(3)(2\delta_1)$ exactly, and 25262·73 which is 29·39 behind is $S_1(3)(-9\delta_1)$. The lines calculated from the formula for $m = 3 \ldots 8$ are given in the accompanying list, which also gives the values as sounded

* R. gives intensity 8, W. gives 3, although as a rule W.'s intensities are higher than R.'s—again pointing to changed displacements with changed conditions of excitation.

from an observed line. Thus in $m = 3$, the observed is entered as $v.(3)$ 38972·12, whereas the actually observed is v less.

m.	Calculated.	Observed.	$d\lambda$.
1		$-\{(2)\ 20163·7\ (W.)+\nu_1+\nu_2\}.v$	*
2		(0) 25432·35 (W.)	*
3	38972·54	$v.(3)$ 38972·12 (R.)	·02
4	43873·80	$e.(4)$ 43874·02 (W.)	−·01
5	46184·76	see Note	·07
6	47454·96		
7	48227·14	$u.v.(1)$ 48227·00 (W.)	·00
8	48731·22	see Note	

Note.—For $m = 3$ the e sounder is too large, and for $m = 4$ the v is too small to act.

The agreement is so good that the chosen low intensity lines for $S_1(1)$ are justified. The low intensity may be explained by the supposition that the energy of the normal line has been partly transferred to other displaced ones, in the way indicated above for $m = 3$ and as shown also by higher orders. For $m = 4$ together with 20195—the line sounded from—W. only observes (2) 20163·7, and R. only (0) 20133·0. These differ successively from the first by 30±. For $m = 4$ the oun displacement produces 1·42 on the sequent and the normal 30·5 on the limit. They are therefore possibly $e.(\delta_1)\,S_1(4)$ and $e.(2\delta_1)\,S_1(4)$. In $m = 5$ the e sounder requires 22506, not observed by either, but there is an isolated group, 22516·9 by both, (5) 22535·5 (W.), (3) 22540·4 (R.), of which 22535 is 30 ahead of the required line and 22540 is 35, so that the former is $(-\delta_1)$ on the limit. The latter, however, sounds to 46218·71 for $(-\delta_1)\,S_1(5)$ and gives $S_1(5) = 46183·25$. The $d\lambda = ·07$ of the list is based on this. For $m = 6$ the e requires 23776·65 with observed (0) 23760·9 (W.) and (10) 23783·75. In $m = 8$ the $u+v$ link $= 24871·8$ requires 23859·5. The only observed are (2) 23841·9 (W.) and (4) 23871·3 (W.) or (5) 70·00 (R.), about equally displaced on either side of the normal line.

Some space has been devoted to the consideration of the S series as on it depends the determination of the important u, v links as well as the limit for the D series. The latter is perhaps in general the more important as it affords with the F, the means of obtaining accurate determinations of the oun. In the present case, however, the measures have such large possible observation errors that they do not add to the accuracy already found in the foregoing discussion. A few points only will therefore be here referred to. Both the D satellites for $m = 1$ and the F lines for $m = 2$ with related lines form the majority of the lines in the long wave end of the spectrum ($n < 20000$). One clue as to a distinction between the two sets may be found in the fact that where the 5640 link occurs on a D line it links backwards, while in the F it links forward. Consequently where a line has a link forward it is probably not a D line, but is either a F line, or is attached to a D line (as in the 1864 set in X attached to the D group near 20312). The D-qualification test with $D(\infty) = S(\infty)$ can be applied to these suspected D lines, taking the 20438 as a definite D_{11} line. That is the mantissa difference of the lines from that of 20438 must be an oun multiple. A very large number are found to satisfy this test. It must be remembered that as the oun is so large as 447 there can be no doubt as to the satisfaction of this test or not, even for the largest allowable observation errors, nor to the actual oun multiple when it is satisfied; on the contrary the possible observation errors are so great, that the observations do not enable us to increase the accuracy of the oun itself as already found. By combining a large number of cases it would no doubt be possible to diminish the probable error of its value, but the heavy work would not be justified at present, especially as there should be hope of better measures in the immediate future. At the same time the existence of the D lines may be illustrated by the following two sets for $m = 1$.

			18357 (C.R.)		(1) 21036·25
			5671 ± 4		**5633·0**
$20\frac{1}{4}\delta$	(2) 18641·4 (W.)	**5387·12**	(1) 24028·52 (W.)	**2640·71**	(1) 26669·23
			19583 (Ram.)		
			5630		
	(1) 19842·4 (W.)	**5371·3**	(2) 25213·7 (W.)		
34δ	(2) 16062 (W.)		(1) 21456·56 (R.)		
	5650		**5633·21**		
	(6) 21712·09 (R.)	**5377·68**	(2) 27089·77 (R.)		
			(0) 16445·61		(0) 19086·1 (W.)
			5652·4		**5662**
	(3) 16725·1 (W.)	**5372·9**	(4) 22098 (C.R.)	**2650·6**	(2) 24748·6 (W.)
$18\frac{1}{4}\delta$	(1) 17493·05 (R.)	**5375·2**	(1) 22868·07 (W.)		
			(1) 18357 (C.R.)		
			5636		
	(0) 18609·94 (R.)	**5383·9**	(10) 23993·82 (R.)		
$31\frac{1}{4}\delta$	(0) 14813 (W.)				
	5625·8				
	(4) 20438·8 (W.)				

In these it should be noted:—

(1) The satellite separations are in the usual ratio 5 : 3, for in the first $20\frac{1}{4} \times 5 = 101{\cdot}25$, $34 \times 3 = 102$ or for D_{23} $20\frac{1}{2} \times 5 = 102{\cdot}5$, $33\frac{3}{4} \times 3 = 101{\cdot}25$. In the second $18\frac{1}{4} \times 5 = 91{\cdot}25$, $31\frac{1}{4} \times 3 = 93{\cdot}75$. Both sets have the same undisplaced limit = 50403, and the two corresponding D_{11} lines are displaced $23\frac{1}{2}\delta$ in their sequences.

(2) The 21712 which acts as a D_{11} line shows the modified ν_1 separation to a second line but of much smaller intensity, in close analogy with what has been seen in X.

(3) The mantissa of $16725{\cdot}1 \pm {\cdot}5$ is $804601 - 26{\cdot}8\xi + 13{\cdot}4p = 25\,(32166{\cdot}16 - 1{\cdot}07\xi + {\cdot}53p) + \delta_1 = 25\,\Delta_2 + \delta_1$. The second line of the triplet 22098 is one of the copper

C.R. lines. Behind this is a line (2) 22079·34 W. ($d\lambda = \pm{}$·05) corresponding to the $-\delta_1$ displacement in the sequent—*i.e.*, the modified D separation. With S (∞) its mantissa = $804161-26·77\xi-26·77d\nu_1+5·35p$

$$= 25\,(32166·44-1·071\xi-1·07d\nu_1+·21p).$$

If this be combined with the mantissa of $29964 = 913165 = 511\delta$ giving $\Delta_2 = 32166·27-1·116\xi$, there results the equation

$$·14+·045\xi+·21p-1·07d\nu_1 = 0.$$

This can be satisfied by $\xi = 0$, $d\nu_1 = 0$, and p a fraction. It does not therefore help to a closer determination. With good measures it should be practicable to find $d\nu_1$ within ·05 and p a small fraction. This equation would then give the small correction for ξ and so increase considerably the degree of accuracy of Δ_2 and δ. The particular point however gained is that here is found one of the fundamental d sequences depending on pure multiples of Δ_2.

(4) They all show -5640 links when this link lands in the observed region except 22868. Where the measures are reliable they congregate round a value 5633 ±. The 22079 of the last paragraph is 5633·73 above the 16445. This is a further justification of 22079 belonging to a $-\delta_1$ displaced sequent.

With 20438 as D_1 (1), RYDBERG's table gives D (2) as in the neighbourhood 37486 ±100. A u sounder gives the region 23806. There is a line (10) 23783·75 (W.) which if linked in this way gives $D_{11}(2) = 37463·75$. The two lines $m = 1, 2$, and S (∞) give the formula

$$n = 50403-\mathrm{N}\Big/\left\{m+·909601+\frac{·003564}{m}\right\}^2$$

with $D(3) = 43242·02$. The e-sounder requires 19563. The only line in the neighbourhood is $\lambda = 5105$ by RAMSAY, who says his measurement is very rough. If we allow $d\lambda = 5$A, the wave-number is 19583±20, and it *may* be the line sought for. There seem also other D_{11} groups as in X. One instance is adduced in the next paragraph.

I end the discussion of the RaEm spectrum by a consideration of the source of the 5640 separation. In X we found the conditions satisfied by oun displacements on the F (∞) of $5\,\Delta_2-\delta_1$, and $2\,\Delta_2+6\delta_1$. But here the values of the separations themselves seem very indeterminate. The values as arranged on p. 431 would seem to point to 5633, 5649 with ν_2 about 2800 and 2820. The limit of the particular F series on which our whole discussion of RaEm is based is 29964·20. As a fact, however, this limit can only generate in the proper neighbourhood separations of 5646, 2806, and the displacements are $5\Delta_2-6\delta_1$, $2\Delta_2+2\delta_1$. The dependence on these displacements so analogous to those in X is evidence that the links in question (5640, 1864) are of analogous type. We have already seen in X that these separations occur as links, as well as direct displacements on sequences present in the wave-number. Let us determine the F limit required to give a separation in the correct neighbourhood with a displacement $5\Delta_2$. The conditions are that not only must the $5\Delta_2$ give the ν_1 but since it must belong to a $d(1)$ sequent, the mantissa of the limit itself shall also be an oun multiple. The result is that the limit should be within close limits of 29617 and that then the ν_1 will be close to 5648 one of the most probable values found on p. 431. But this $29617 = F(\infty) = d(1)$ should belong to a D line $50403-27917 = 20786$. Now there is an observed line (0) 20784·31 (W.) with another at (7) 20750·34 (W.) or (7) ...52·70 (R.) behind it. Taking 20784·31 its sequent is 29618·69 and mantissa 924298 or 11133 above that of 29964·20. Now $6\frac{1}{4}\delta = 11169$ and the difference 36 requires a change in wave-number $dn = 1{\cdot}10$ or $d\lambda = -{\cdot}25$A. The line was only read to ·1A so that this error is permissible. The $F(\infty)$ is then 29617·57. The $-5\Delta_2$ displacement on this gives 35266·16 or a separation 5648·59, and on this it will be found that an extra $2\Delta_2+5\delta_1$ gives $\nu_2 = 2810{\cdot}97 = 2811$ say. Further a change of δ_1 on $5\Delta_2$ or $2\Delta_2+5\delta_1$ produces changes in ν_1, ν_2 respectively of 17·88 and 20·00. This accounts for the concomitant value of 5630·71, and 2831 also observed is due to $5\Delta_1-\delta_1$, and $2\Delta_2+6\delta_1$ precisely those in X.

The Value of the Oun.—The data for the evaluation of the oun are :—

(1) The triplet separations and $S(\infty)$. The ν_1, ν_2 are not determinable directly from S lines and consequently no exact values can be obtained from observation. The general argument on p. 433 is in favour of 5371, 2649, or about 3 larger according to limit. These are strongly supported by the set of lines on p. 437, clearly a linked S (1) or an independent S (1) set showing also the displaced S_2, S_3 by W. and R. respectively. These values point to $\delta = 1787{\cdot}0$.

(2) From the F and **F** series giving definite $F(\infty) = 29964{\cdot}20$ and $f(1)$; p. 430. The definite result is a limitation of δ to one of six alternative values, of which four between 1785·26 ... 87·87 are equally probable, the ambiguity being due to imperfect measurement. This argument is independent of the values of ν_1, ν_2.

(3) The mantissa of the D_{13} line 16725 is $25\Delta_2+\delta_1$ and that of the D_{23} line 22079·34 is $25\Delta_2$. This gives

$$\Delta_2 = 32166{\cdot}44-1{\cdot}07\xi-1{\cdot}07d\nu_1+{\cdot}21p,$$

$$\delta = 1787{\cdot}024-{\cdot}059\xi-{\cdot}059d\nu_1+{\cdot}012p.$$

Here probably

$$\xi<{\cdot}5, \qquad d\nu_1<{\cdot}1, \qquad p<{\cdot}5, \text{ or, say,}$$

$$\delta = 1787{\cdot}024\pm{\cdot}05.$$

These three determinations are all independent.

Argon.—There are a large number of strong lines in the visible spectrum evidently connected with D(1) and F(2) lines. The oun is so small that the qualifying test for D lines is not so definite as in the other gases, although this is to some extent remedied by the fact that the measures on the whole are good with a possible maximum error of ·02A. We shall therefore make no attempt to discuss the F and D series with the same fullness as in the other cases. The groups selected are certainly not the only ones and possibly may not be the most important ones, but they will be sufficient to give data for the determination of the oun to about the same degree of accuracy as for the other gases of this family. Take for the first group.

	$m = 1.$			$m = 3$	
(1) 23782·51 (K.)	**179·92**	(6) 23962·43 (E.V.)	(1) 44827·37)	**175·51**	(1) 45002·88
$71\frac{1}{2}\delta$			$39\frac{1}{4}\delta$		
(2) 23899·08 (K.)			(3) 44835·41		

	$m = 2$	
(3) 39357·06	**178·95**	(4) 39536·01
$171\frac{3}{4}\delta$		
(3) 39420·68		
$39\frac{1}{4}\delta$		
(5) 39439·33		

The lines for $m = 2, 3$ are by Eder and Valenta. Again the low intensity for $D_{11}(1)$ is to be noticed, but we have here some indication of the source of this peculiarity. Eder and Valenta give the line at 23899·83 of intensity 9, and state that it appears only with a very strong condenser discharge. The difference in the two measures ($d\lambda$ = ·13A) may possibly be due to observation although it is greater than is the rule between the measures of K. and of E.V. As an oun displacement in the sequent produces a change dn = ·4, the difference may be due to $2\delta_1$ displacement. If so, no error will be introduced in the succeeding considerations by overlooking this, treating it as due to observation error and using Kayser's measurements, subject to a smaller possible error. For in either case the dependence of the sequent mantissa on the multiple law is not affected and although the multiple itself is different, at the same time the difference in the two readings will not modify the formula constants to an extent to appreciably affect the calculated lines for $m > 2$.

Using the limit $D(\infty) = S(\infty) = 51731·03$ and the first two D_{11} lines the formula found is

$$n = 51731·03 - N \Big/ \left\{ m + 989074 - \frac{·003976}{m} \right\}^2 .$$

This gives for $m = 3, 4, 5$, the lines $n = 44834{\cdot}16$, $47323{\cdot}04$, $48672{\cdot}56$. The O−C for the first is $d\lambda = -{\cdot}06$ whilst the others should be weak and are near to the limits of observations. The links are so small that as sounders they produce lines also close to the observed region. For $m = 1$, the difference of the sequent mantissæ of 23899 and 23782 (D_{12}) is 4144 and $4\Delta_2 - 6\delta_1 = 71\frac{1}{2}\delta = 4141{\cdot}2$, or exact with $d\lambda = {\cdot}013$ on two lines. It will be shown later (p. 447) that D_{12} depends on the Δ_2 multiple. The series is, therefore, probably a real doublet series with this for the outer satellites. For $m = 2$ the D_{13} set with a displacement of about $171\frac{3}{4}\delta$ is inserted since it recalls the order of magnitude in Kr and X, whilst the two D_{12} satellites for $m = 2$ and 3 are inserted because they show the same displacement of $39\frac{1}{4}\delta$. An extrapolated line on an observed one [23611·5], **179·5**, (2) 23791·0 gives a displacement 175δ for $m = 1$ corresponding to that for $m = 2$. It may also be noted that in Argon the third lines of the triplets appear to be missing in this D series.

The following arrangement would seem to indicate the existence of a set parallel to the above:—

23899·08	**20·74**	(3) 23919·82	**188·73**	(1) 24108·55
39439·33	**20·65**	[39459·98]	**171·60**	(3) 39631·58
44835·41	**20·65**	[44856·06]	**177·22**	(4) 45033·28
[47323·01]	**20·66**	(47343·67)	**178·97**	(1) 47522·64

The [ ] in the second column are calculated from the first column by adding 20·65. The line in ( ) is determined as the mean of the linked lines $u.$(1) 47784·02 = ... 44·55 and (1) 46183·62.$u.e$ = ... 42·80. As 5δ on the limit produces a separation of 20·65, the lines in the second column would correspond to the parallel displaced series $(-5\delta)\,D_{11}$. The separations of the last column increase with the order to a normal value of ν_1 and point to a constant displacement in the sequent.

With the above D set should be associated a doublet F series whose $F_1(\infty) = d_{11}(1) = 51731{\cdot}03 - 23899{\cdot}08 = 27831{\cdot}95$ and separation that of the D satellite or 116·5. In searching for this allowance will have to be made for the prevalence of displacement in the lower orders. As a fact, however, this only appears for $m = 3$ in F_1, although in F_2 there are considerable signs of it. The following table with succeeding notes contains the data, including also those for the corresponding **F** series; also for certain lines which show a separation about 150, and which may probably be the analogue of the 1864 X series.

We shall denote this last set by dashing the letters.

The formula obtained from the two first F_1 lines and the given limit is

$$n = 27831{\cdot}95 - N\Big/\left\{m + {\cdot}757701 + \frac{{\cdot}006544}{m}\right\}^2.$$

The O – C between observed (or deduced from observed) and calculated are given under the heading $d\lambda$. As before ( ) refers to deduced lines [ ] to calculated.

m.	F.	$d\lambda$.	27831·95.	F.
1	– (4) 7404·33 (P) **115·65** – (4) 7288·68 (P) ...	*		[63066] ... ...
2	(1) 13444·52 (P) **116·12** (1) 13560·64 (P) **150·91** (1) 13595·43 (P)	*	*31·42*	(42218·33) **115·89** (42334·22) ...
3	[20073·40] **116·85** (20190·25) **151·05** (4) 20224·41	?	*31·14*	(35588·89) **114·3 ± 2** (35703·2 ± 2) ...
4	[89·50] (10) 22991·57	– ·39	*30·13*	(32668·69) **116·31** (32785·0) **150·50** (2) 32819·19
5	[24·49] (5) 24521·94 **115·31** (24637·25) ...	·42	[*30·88*] *29·60*	(1) 31137·27 **116·57** (31253·84) **150·29** (31287·56)
6	(2) 25429·28 **116·54** (25545·82) **152·84** (1) 25582·12	·17	32·12	(1) 30234·96 **116·85** (4) 30351·81 ...
7	(26010·63) **116·44** (26127·07) ...	– ·19	33·29	(3) 29655·76 ... ...
8	(4) 26402·01 **114·27** (26516·28) **147·75** (5) 26549·76	– ·07	30·99	(29259·97) **118·82** (29378·79) ...
9	(1) 26679·62 ... **153·75** (1) 26833·37	– ·01	34·96	(3) 28990·30 **119**·00 (29109·30) **150·51** (1) 29140·81

m.	F.	$d\lambda$.	27831·95.	**F.**
10	(5) 26885·78	–·29	34·10	(1) 28782·42
	...			... **148·23**
	...			(1) 28930·65

Notes.—Displacements are in evidence except for F_1. A desired wave-length can be obtained very closely since a δ_1 displacement produces a change of only ·42. Consequently in deductions by displacement the results have little weight, and in fact would have none at all were it not that we now know as a fact that such exist. Failing such likely displacements, recourse has been had in a few cases to linked lines and here their evidence has weight.

$m = 2$. For $\mathbf{F}_1$, (-16δ) (1) 42192·13 = 18·21; (9δ) (1) 42233·12 = 18·45; mean taken. For $\mathbf{F}_2$ (-5δ) (1) 42326·07 = 34·22.

$m = 3$. There is no observed F_1, but (3) 19351·20.*e* = 20070·91 or with E.V. 71·65;

F_1 (3) $(7\Delta_2)$ = 20103·85 and 20105·52 is observed by E.V. $d\lambda = -\cdot 42$.

For F_2, $(-38\delta_1)$ (4) 20174·83 (E.V.) = 90·22; $(20\delta_1)$ (2) 20198·39 = 90·29 mean taken.

For $\mathbf{F}_1$, (-5δ) (1) 35580·73 = 88·88, also $\mathbf{F}_2$, (-5δ) (1) 35695·03 E.V. ± 2 = 703·18 ± 2.

$m = 4$. The observed line for F_1 is much stronger than should be expected; also its O – C is in the opposite direction to that of the others. It may be displaced by Δ_2 on the sequent and so intensified. The Δ_2 would produce O – C = ·00 and the observed = F_1 (4) (Δ_2), or it may hide the real F_1. $\mathbf{F}_1$ is inserted as 32819 – 150·50. It only indicates that 32819 is the correct value of $\mathbf{F}'$. $\mathbf{F}_2$ is given as (5δ) (1) 32793·11.

$m = 5$. F_1 shows a link *e* = 719·81 to (3) 25241·75. F_2 is entered as (18δ) (4) 24665·59 = 37·25, but has no weight. $\mathbf{F}_1$ = $(15\frac{1}{2}\delta)$ (1) 31279·10.

$m = 6$. F_2 is the mean of $(-6\delta_1)$ (2) 25536·15 = 45·87 and (7δ) (1) 25557·12 = 45·78. The calculated value of F_1 gives a better 151·7 to F'_1.

$m = 7$. The calculated F_1 is 26009·30. The deduced value is (1) 25290·92.*e* = 26010·73, which again has an apparent approximate *e* link = 720·77 forward to an E.V. line at (2) 26731·40. This is however a coincidence as the last line is S_2 (2). In connection with the linked line 26010·73 may be taken the pair of lines (-6δ) (3) 26001·11 = 10·89 and $(7\frac{1}{2}\delta)$ (3) 26022·85 = 10·63 whose mean agrees precisely with the former. For F_2 a similar split with two may be observed $(-2\frac{1}{2}\delta)$ (3) 26123·03 = 27·07 and (2δ) (1) 26130·61 = 27·30.

$m = 8$. F_2 is (5) 25796·57.*e*; also (-2δ) (2) 26512·75 = 16·05. It may be noticed that the separations to F_1 and F' are both about 2·3 too small, or $1\frac{1}{2}\delta$ on the limit. $\mathbf{F}_1$ is (2) 28540·26.*e* and $\mathbf{F}_2$ is (1) 28659·08.*e*. These linked lines in this order are therefore reliable.

$m = 9$. $\mathbf{F}_2$ is (1) 28389·59.*e*.

It should especially be noted how with increasing order the normal calculated lines appear as observed with good intensities. As in the other instances it suggests itself that this is due to the diffusion of the energy of the lower order lines into the formation of numerous displaced ones. The usual accurate determination of the limit as the value of $\frac{1}{2}(F_1+\mathbf{F}_1)$ is not here applicable as, for the reason given above, the determination of the actual displaced lines is unreliable. These values are printed in italics. The more reliable results point to a limit higher than that calculated in $S(\infty)$, with ξ about +2.

The Value of the Oun.—The preceding results for the D and F series afford material for the more exact determination of the oun. The D_{11} line 23899 must have a mantissa which is a multiple of the oun, though not necessarily of Δ_2. The mantissa of the $F(1) = -7404$ should be a multiple of Δ_2. PASCHEN'S estimate for the maximum error of the F(1) is 1A, and KAYSER'S for the D(1) is ·02A. These mantissæ are, introducing the actual errors as $d\lambda = p_1$, $·02\,p_2$

$$985098 + 4·06p_2 - 35·66\xi$$
$$764245 + 13·7p_1 - 25·03\xi = 723\,\{1057·047 + ·0190p_1 - ·0346\xi\}.$$

In the case of F(1), given that its mantissa is a multiple of Δ_2 the multiple must be 723 for 7.24 or 725 would alter Δ_2 by 1·5, whereas we already have its value as close to 1057·0. The mantissa of d_{11} is so large that by itself it is not possible to determine the oun multiple. But the difference of the mantissæ of d_{11} and f is much smaller and also an oun multiple. It is

$$220853 - 13·7p_1 + 4·06p_2 - 10·66\xi$$
$$= 209\,\{1057·047 + ·0190p_1 - ·0346\xi\} - 5\delta_1 + 2 - 17·7p_1 + 4·06p_2 - 3·67\xi.$$

Even this is too large to settle the multiple on account of the observation errors and uncertainty in ξ. It may be an oun more or less. The following consideration, however, will give some indication on this point. The mantissa of the satellite 23782 as found above was $4\Delta_2 - 6\delta_1$ above that of D_{11}. Consequently its complete mantissa is $209\Delta_2 - 5\delta_1 \pm \delta_1 - (4\Delta_2 - 6\delta_1) = 205\Delta_2 + \delta_1 \pm \delta_1$ above that of $f(1)$ or $= 928\Delta_2$ or $+\delta_1$ or $+2\delta_1$. Now this is so close to the Δ_2 multiple as to point to the fact that this satellite is the fundamental one, in which the rule is a multiple of Δ_2 for the mantissa when there is no relative displacement with the second or third of the triplet satellite set. But here the observed ν_1 is $179·92 = 179·50 + ·42$ and ·41 is the change produced by an oun displacement. In other words the mantissæ of the D_{12}, D_{22} lines are either $M(\Delta_2)$, $M(\Delta_2) + \delta_1$, or $M(\Delta_2) - \delta_1$, $M(\Delta_2)$. But it cannot be $M(\Delta_2) - \delta_1$. Hence if the multiple law holds here it must be in the D_{12} satellite, the $-\delta_1$ must be taken above, or the difference is $209\Delta_2 - 6\delta_1$ and $16·4 - 17·7p_1 + 4·06p_2 - 3·69\xi = 0$. This is easily satisfied by moderate values of the p's and ξ. Further, the mantissa of 23782 is $928\Delta_2$.*

This makes the mantissa of $D_{11} = 932\Delta_2 - 6\delta_1$, whence

$$932\Delta_2 = 985098 + 6\delta_1 + \ldots = 985184·9 + 4p_2 - 35·66\xi$$
$$\Delta_2 = 1057·064 + ·004p_2 - ·0382\xi.$$

* It may be noted in passing that the satellite separation is 116·57, but is reproduced in the F series as 115·65 (P.) for $m = 1$, 116·12 (R.) for $m = 2$. The latter, more reliable, is ·45 less than the corresponding satellite separation. This general effect is therefore completely explained by the F limit for the second series being d_{22} in place of d_{12}.

The D_1 satellite gives

$$928\Delta_2 = 980956+p_3-35{\cdot}44\xi, \quad \text{obs. error} = {\cdot}02p_3$$
$$\Delta_2 = 1057{\cdot}064+{\cdot}04p_3-0382\xi.$$

Also the $f(1)$ gives

$$\Delta_2 = 1057{\cdot}047+{\cdot}0190p_1-{\cdot}0346\xi.$$

The value of ξ in $f(1)$ is not precisely the same as in the others as it involves the observed error in D_{11}. If ξ is the value for $d_{11}(1)$ that for $f(1)$ is $\xi-{\cdot}12p_2\,(d\lambda = {\cdot}02p_2)$ and

$$\Delta_2 = 1057{\cdot}047+{\cdot}0190p_1+{\cdot}004p_2-{\cdot}0346\xi.$$

Thus p_1 is of the order ·9 and the best value is

$$\Delta_2 = 1057{\cdot}064-{\cdot}0382\xi\pm{\cdot}004.$$

The value of ξ is probably a very small positive quantity. The O−C for $D_{11}(4)$ is $dn = 1{\cdot}25$ pointing to a positive ξ. The data for the accurate determination from $\frac{1}{2}(F+\mathbf{F})$ are defective. Of directly observed lines those for $m = 6$ appear reliable as the separations are good. This gives the limit as 27832·12 or $\xi = {\cdot}17$. Failing any better determination we put $\xi = {\cdot}17+\xi$

$$\Delta_2 = 1057{\cdot}057-{\cdot}0382\xi\pm{\cdot}004. \qquad \delta = 57{\cdot}9209-{\cdot}0021\xi\pm{\cdot}0002.$$

Probably

$$\xi \not> 1 \qquad \text{and} \qquad \delta = 57{\cdot}921\pm{\cdot}002.$$

The manner in which so many independent conditions fit in to give this value is very remarkable, and should give great weight to the accuracy of the above value. It must not, however, by itself be regarded as a definitive value free from all possible doubt without further support from independent groups of D or F series. This is not easy to obtain. In the first place the spectrum of A seems to approximate more to the doublet sets of He, and we lose the advantage of dealing with triplets. Again, it is clear from the way in which the lines are crowded in the region below $n = 2300$ and in which modified separations in the neighbourhood of 180 preponderate that a large number of displaced groups must exist, and displaced groups have generally shown a fragmentary quality. It is difficult, however, to deal with these in the same way as in Kr, X, or RaEm, because the oun is so small that the separations produced by one oun displacement are comparable with observation errors or uncertainties in the limits.*

* I had prepared the details of an additional D group with corresponding F and **F** series depending on a displaced $D(\infty)$. As however, while supporting the above value of the oun, it does not decrease its possible error, no present object is to be gained by printing it.

The separation 150 has been referred to as probably the analogue of the 1864 F separation in X. The reasons for this supposition are based on its magnitude and its occurrency curve. This separation in Kr, X and RaEm has been explicable as due to a displacement close to $5\Delta_2$ on the mantissa of a $d(1)$ sequent. In the present case the example taken is not the most important F series, but its limit will be near that to which the separation is due. The displacement required in it to produce a separation 150 is very close to $5\Delta_2$, in fact $5\Delta_2$ produces 149. The separation in question then is caused by displacement of the normal Δ_2-multiple for this series. The second reason is based on the form of the occurrency curve, which shows the same sharply defined single peaked curve as in X. It is represented in Plate 2, fig. 5.

It may be interesting to note that all STARK'S A^{+++} lines, *i.e.*, whose sources have lost three electrons, all show e links except one. They are

			(3) 23674	**719·23**	(3) 24393	
			(3) 23696			
			(2) 24053	**719·22**	(4) 24772	
721·90	(1) 23639	**719·46**	(7) 24359			
		? F_1 (5).	(5) 24521	**719·81**	(3) 25241	**720·24**.

Neon.—The principal sources* for measurements in the spectrum of Neon are observations by LIVEING and DEWAR, BALY, and WATSON. These have been supplemented by interferential measures by PRIEST, MEGGERS, and MEISSNER. ROSSI has added a few lines down to 2352. Through the kindness of Mr. W. F. MEGGERS I have also had the advantage of using an as yet unpublished list of very accurately measured lines made by himself and Messrs. BURNS and MERRIL at the Bureau of Standards in Washington.† The lines by LIVEING and DEWAR are only roughly measured, but as in the case of the other rare gases comprise many not observed by others. BALY'S list extends from 6717 to 3037, WATSON'S from 7245 to 2736 and contains a considerably larger number of lines. Both in BALY'S and WATSON'S lists considerable gaps appear with only a few strong lines, especially between 4250 and 3500. These are filled by a number of weak lines observed by LIVEING and DEWAR. These latter are very important for the complete discussion of the Ne spectrum as they represent the scattered displacements and linked lines of low order series lines which normally should occur as single strong lines but which are here wanting.

* LIVEING and DEWAR, 'Roy. Soc. Proc.,' vol. 67, p. 467 (1900); E. C. C. BALY, 'Phil. Trans.,' A, vol. 202, p. 183 (1903); H. E. WATSON, 'Roy. Soc. Proc.,' A, vol. 81, p. 181 (1908); J. G. PRIEST, 'Bull. Bur. Standards' (U.S.A.), vol. 8, p. 2; W. F. MEGGERS, 'Bull. Bur. Standards,' vol. 12, p. 198 (1915); K. W. MEISSNER, 'Ann. d. Phys.,' vol 51, p. 115 (1916); R. ROSSI, 'Phil. Mag.,' vol. 26, p. 981, (1913).

† Referred to below as B.M.M.

Unfortunately L.D.'s measures are only given to 1A, and thereby their value is greatly diminished as they become merely indicatory and cannot serve as quantitative data. The accuracy of BALY and of WATSON is good and probably about the same. PRIEST claims an accuracy with probable error $<$·0005A, MEISSNER with error not $>$·0015, but the accuracy of an interferometer measure depends very largely on the nature of the individual line. MEGGERS' results are exceptionally valuable in that he gives interferometer measures of a number of lines of small wave-length 3701 to 3370 where the S (2) and some of the higher order F lines occur. ROSSI has succeeded in allotting lines to series.

Neon affords an apparent exception to the rule amongst the rare gases of different spectra, according as they are developed with or without condenser in the tube discharge. On the other hand its spectrum would appear to be a composite one of the typical "red" and "blue" spectra. It undoubtedly has a portion analogous to the "blue" as will be seen by the results obtained below, completely analogous to those found in this communication for the other gases, which refer to their "blue" spectra. On the other hand in some remarkable sets of accurately equal separations discovered by WATSON* it shows a relation to the analogous well-known constant separations observed by RYDBERG in Argon. Further, it is specially rich in lines in the red region. In the list of lines observed at the Bureau of Standards referred to above there are 225 between 8783 and 5689. Since in each periodic group of elements the number of lines as a whole increases very rapidly with the atomic weight, the excess of red lines in Ne is even comparatively greater than the actual number shows. The majority of these lines are weak, but they almost all fall into a few definite linkages in which the links are the constant separations discovered by WATSON. Some of these special linkages again are connected together by the p and s links, especially the *e.u.v.* They belong to the F type of order $m = 2$, and should afford most valuable information as to the way in which parallel and displaced lines are related. I hope to return to this question on a later occasion, and only refer to them in the present discussion incidentally as they afford some evidence for the determination of the value of the oun.

The wave-numbers of observed lines published stretch from 13251 to 36536. From analogy with the spectra of the other gases we must not therefore expect to find more than one order in each of the S and D series. Nor, with its small atomic weight will the *e.u.v* links be large enough to act as efficient sounders. On the other hand the whole of any F series ($m = 1$ excepted) should lie within the above limits. It is therefore clear that the attack on the problem must be made first on this series. One datum at least is at our disposal in the magnitude of the oun. Taking the atomic weight at 20·0 ± ·01 the calculated value of δ is 14·47 ± ·01. This value of 14·47 may therefore be treated as exact to one or two units in the last digit.

* 'Proc. Camb. Phil. Soc.,' vol. 16, p. 130 (1911); 'Astro. Journ.,' vol. 33, p. 399 (1911).

The F (2) line should be a strong one in the neighbourhood of 17000. There are a number in this region. The lines in the following list are found to give a good series and is doubtless *one* F series. In this case we possess the great advantage of very accurate interferometer measure of the 1st, 3rd, and 4th lines in international units. These are used to determine the formulæ constants. The wave-lengths are given as measured, the wave-numbers are all in Rowland units.

NeF.

m.	λ.	*n*.	O.	O – C.
2	(10) 5852·48 I.	17081·46	·00	*
3	(0) 4157·74 R.	24044·88	·05	– ·05
4	(5) 3700·89 I.	27009·47	·00 ?	*
5	(5) 3501·22 I.	28552·37	·00	*
6 {	3393 R.	29464·2	1·00	– ·66
	(3417.*e*) R.	(29454·2)	1·00	(·47)
7	3329 R.	30030·57	1·00	·58

Notes to Table.—For the first line PRIEST gives λ = 5852·4862, MEISSNER ·4875, and MEGGERS ·488. These all give the same wave-number to the second decimal place. The second is a weak line by WATSON not observed by BALY. BALY gives a line, intensity 4, $n = 24039·45$ not observed by WATSON. We have here a concrete observational example of the facility with which a normal F line of low order can split up into displaced lines by slightly different excitations. In this case the mantissæ difference in the sequents is 159 and $11\delta = 159·1$, so that 24039 is F (3) (-11δ). The third line is a strong line observed by both WATSON and BALY, but the measure used is deduced from an interferentially measured line $n = 28439·801$ by deducting the WATSON link separation 1429·429 (both accurate). WATSON'S measure is 27009·95. The line for $m = 5$ is by MEGGERS, but WATSON gives the same *n*. The remainder of the series comes in one of the gaps referred to above in which only L.D. have observed. They give lines which may serve for $m = 6, 7$. Also for $m = 6$ there appears a linked line at $29257·2+e = 29454·2$ (using the value of *e* found below). The calculated wave-numbers for $m = 8, 9, 10$ are 30422·6, 30703·3, 30906·5. No lines are observed between 30203 and 30722. The last two have lines by L.D. near them at $30722·7 \pm 9$, $30922·3 \pm 9$.

The formula as found from $m = 2, 4, 5$ is

$$n = 31850·19 - \mathrm{N} \Big/ \left\{ m + ·794726 - \frac{·139254}{m} \right\}^2 .$$

The calculated value for F (1) is λ = 12241·97 *in vacuo*.

There can be little doubt about these lines forming a series, but there is so far no independent evidence that it is of the F type beyond its analogous position in the spectrum to that of the other rare gases. If it is of this type we should expect to find a number of parallel series as well as of the associated **F** type. It is then necessary to test for these conditions.

A considerable number of parallel series can be arranged all weak for $m = 3$ as in the original series. Some of these are given in the following list in which the numbers in clarendon below the wave-numbers give the separations from the corresponding F lines. Those to the right of the vertical line are added. The others are deducted.

(1) 16768 {2·67 (0)...71; 5·56 (0)...74} **312·73**	(8) 16816 **265·36**	(2) 16831 **249·59**	(2) 16845 **235·84**	(4) 16889 **191·63**
23623 **315·2 ± 3**	—	(0) 23798 **246·69**	(1) 23812 **232·58**	(2) 23854 **191·29**
26695 **314·6 ± 3**	*26745* **264 ± 3**	*26766* **244 ± 3**	(1) 26776 **233·33**	*26816* **193 ± 6**
	u.(9) 28396 **262·83**	(1) 28302 **250·28**	*28216.u* **228 ± 6**	—
	(3) 29196‡ **261·7**	—	—	*29257* **200 ± 9**

(5) 16925 **155·86**	(1) 16948 **113·00**	(5) 16996 **85·23**	|	(5) 17222→ **141·51**	(7) 17342 **261·22**
—	—	—	|	*24183* **138 ± 6**	*24312* **267 ± 6**
26860 **150 ± 6**	(26892)* **117 ±**	(2) 26922 **87·10**	|	(4) 27148→ **138·83**	*27285* **275 ± 6**
(9) 28396 **156·33**	(5) 28438 **113·61**	(4) 28474 **87·57**	|	*e*. (5) 28888 **138·68**	(28819)† **267 ± 6**
(3) 29196.*u*‡ **155·0**	*29343* **115 ± 9**	*29369* **89 ± 9**	|	(1) 29592 **134·15**	*29727* **268 ± 9**
			|	*30175* **139 ± 9**	*30194.u* **265 ± 9**

The wave-numbers in italics are by L.D. and subject to probable errors $d\lambda = \cdot 5$ and possible $d\lambda = 1$. For values of links see below.

* *u.26997·6* = 26891·0; *266948.e* = 26891·8.

† *u.28927* = 28820·3; *28711.u* = 28817·7, mean taken.

‡ One at least must be a coincidence, *or* the two series can simply be linked by *u* or *v*.

Still more striking, and as will be seen later, important, are parallel series with the separations discovered by WATSON. In addition are found also others at 1932 behind F (2). They are

(5) 15149·24	**1932·22**	F (2)	**1429·38**	(6) 18510·84	**422·33**	(1) 18933·34
22103	**1941·7 ± 5**	F (3)		see Note*		
25087	**1922·9 ± 6**	F (4)	**1429·43**	(6) 28438·85	**417·44**	(4) 28856·29
(3) 26628·53	**1924 ±**	F (5)	**1426 ± 8**	*29976·5*	**424**	*30203.e*†

* F (3) has a link 423 to *24467* ± 5 and then 1432 to *25899* ± 5 suggesting a mesh in which the required line is wanting. Here the 1932 separation goes better with the strong displaced F (3) 24039·45, giving 1936 ± 5. The mesh should be

F (3),	24044	**423 ± 6**	*24467*	**1432 ±**	*25899*
		1429·42	[25474·30]	**425 ± 6**	

† Note that 1426 + 424 = 1429·4 + 420·6 ± 9.

Any lines of the **F** type up to $m = 4$ will unfortunately lie in the ultra-violet beyond the observed region. **F** (4) should be 36690, and the largest frequency observed by WATSON is (1) 36536. The others should be weak and in a region where glass apparatus would only allow strong lines to be registered. **F** (5) should be at 35148·01 + 2ξ but is not seen. The line 35259·2 is about a v link ahead, in fact v.35259·2 = 35152·4. It may, however, be noticed that 36536 above is just 154 behind the expected **F** (4), so that it is the **F** line corresponding to the parallel F set above with the separation 156 (say F′). In the same series is also found **F′** (6) = (2) 34087·1 corresponding to the F′ (6) = 29196.u. These are of value in that it gives the means of determining the limit with great exactness. Denote the parallel series by F′. For $m = 2$ using B.M.M.'s measure for F′ (2) the separation is 17081·46 ± 0 − 16925·43 ± ·05 = 156·03 ± ·05. For $m = 5$ both lines have been measured interferentially and the separation is 28553·342 − 28397·167 = 156·175, correct to the second decimal place. The two separations differ by more than the allowable observation error, and is possibly due to the common change in sequent for series with different limits. In these cases in the separation with the larger m, this effect is very small. Consequently we are justified in taking the separation as 156·17 ± 0. For $m = 4$ we have F (4) and **F′** (4) but only a L.D. line for F′ (4). The separation, however, gives its exact value as 27009·47 − 156·17 = 26853·30. **F′** (4) is 36536·62 ± ·66 ($d\lambda$ = ·05). The mean gives the limit for the F′ series as 31694·96 ± ·33 and consequently for F as 31851·13 ± ·33, *i.e.*, ξ = ·94 ± ·33.

But further in the neighbourhood of calculated **F** (6) = [34242] are found also (1) 34336·06, (3) 33918·08 respectively 94 ahead and 323·9 behind it. In analogy also are found (5) 17176·34, (3) 16757·91 respectively about the same amounts ahead of and behind F (2), but no other corresponding F (m) lines appear. We are justified in taking the first two lines as really in the relation indicated, and are thus enabled to arrive at the value of F (6). Using B.M.M. measures the separations given by the F (2) lines are their differences from 17081·46 or 94·88, 323·55 with very small errors ±·05. These, therefore, give for **F** (6) 34336·06±·25−94·88 = 34221·18±·25 and 33918·08±·5+323·55 = 34221·63. Both are therefore within error limits of their mean 34221·40. With this for **F** (6), and the limit corrected as above to 31851·13 the value of F (6) is 29460·96. With the value calculated from the formula O−C = −·23.

As illustrating the way in which D and F sequents are subject to displacements, it is interesting to notice that although the separations 94, 323 do not appear as directly dependent on F lines after $m = 2$, they nevertheless occur in the neighbourhood. It will be sufficient here only to refer in detail to the case of F (4) = 27009·47. At about 50 ahead of this there is a line (6) 27060·60 (W.). With this there is the following scheme :—

26766·3 (L.D.) **96·6** 26863·4 (L.D.) **197·2** (0) 27060·60 (W.) **321·4** 27382 (L.D.)

As the L.D. lines are subject, even if correct to the nearest A.U., to equally probable errors between $d\lambda = \pm\cdot 5$, or here to $dn = \pm 4$, these separations correspond to the 94·88, and 323·55, whilst 197·2 is the link *e*. The displacement of 27060 from F (4) may then be in the F sequent or the limit—probably the former, or *a*, *b* or *c* link.

The existence of the parallel sets, the indications of the associated **F** types, and the nature of the displacements all point to the conclusion that our original series is of the F type. Consequently the limit F (∞) = 31851·13 is a *d* (1) sequent, but there is nothing as yet to show whether it belongs to a d_{11} or a satellite set. If, however, we can find a D (1) set the value of D (∞) can be obtained with sufficient accuracy to obtain the values of Δ_1, Δ_2 and the *e* link. The further consideration of the F series will therefore be postponed until this further information has been obtained.

It has already been remarked that only the S (2) and D (1) lines in any S or D group can be expected within the observed region. A superficial inspection of the list of lines shows a very large number of separations in the neighbourhood of 46 to 49 and about 20, clearly related to ν_1, ν_2 values as they show the normal ratio $\nu_1 : \nu_2$, and are in step with those of the other gases. In about the region in which the D lines should be expected BALY gives the strong set (4) 21200·90 **49·24** (4) 21250·14 with an equally strong line at (4) 21230·11 which might be a D_{11} line to the doublet D_{12} set. WATSON, however, gives other strong lines as well, including a triplet. These give

(1) 21156·77 (W.)	**48·18**	(1) 21204·95 (W.)	**19·71**	(4) 21224·66 (B., W.)
44·13		**45·19**		
(4) 21200·90 (B.)	**49·24**	(4) 21250·14 (B.)		
(5) ·81 (W.)	**49·36**	(5) ·19 (W.)		
29·21				
(5) 21230·11 (B., W.)				

We are justified by its form in taking this as a D (1) set for a preliminary trial, although the satellite separations are not in the usual ratio of 5 : 3. It is not so clear that the particular F (∞) just obtained is a d sequent belonging to it, nor, if so, whether it is a d_{11} or a satellite sequent. The latter point, however, will have little effect for our immediate object--the attainment $\Delta_1 . \Delta_2 . e$—as the differences are small and may be included in an undetermined ξ. The probability is that as this seems the only prominent D triplet set, it belongs to the normal group and that our F (∞) belongs to it, although on this point something will have to be said later. We shall take on trial that $F(\infty) = d_{11}$. In this case

$$D_1(\infty) = 21230{\cdot}11 + 31851{\cdot}13 = 53081{\cdot}24 + \xi$$

This should also be $S_1(\infty)$ and should give the separations by oun multiples in the denominator of the sequent. The three S or D limits would then be

$$53081{\cdot}24 + \xi \quad \mathbf{49{\cdot}24} \quad 53130{\cdot}48 + d\nu_1 + \xi \quad \mathbf{19{\cdot}71} \quad 53150{\cdot}19 + d\nu_1 + d\nu_2 + \xi$$

where ξ *may* be considerable, owing to uncertainty as to 31851 being of d_{11} or satellite type. The mantissæ of these are

$$437428 - 13{\cdot}540\xi, \qquad 436752 - 13{\cdot}521\xi, \qquad 436485 - 13{\cdot}514\xi$$

which give as differences

$$\Delta_1 = 666 - {\cdot}019\xi + 13{\cdot}5 d\nu_1 = 46\,(14{\cdot}477 + {\cdot}29 d\nu_1 - {\cdot}0004\xi)$$
$$\Delta_2 = 267 - {\cdot}007\,(\xi + d\nu_1) + 13{\cdot}5 d\nu_2 = 18\tfrac{1}{2}\,(14{\cdot}432 + {\cdot}73 d\nu_2 - {\cdot}0004\xi),$$

in which it must be remembered that calculations with seven-figure logarithms give uncertainties of a unit in the last digit. The direct calculation of δ from the atomic weight has already given $\delta = 14{\cdot}47 \pm {\cdot}01$. This gives $46\delta = 665{\cdot}6 \pm {\cdot}46$, $18\frac{1}{2}\delta = 267{\cdot}69 \pm {\cdot}18$. Thus these limits give without any doubt the true oun multiples in Δ_1, Δ_2, and the calculated Δ_2 then gives a closer value of $267{\cdot}7 \pm {\cdot}2$. No possible change in ξ can affect these results. Further, these multiples are quite in step with the march in the other gases. The remarkably close agreement sustains the allocation of the F (∞) to the d sequence of this set, although not necessarily to d_{11}.

As a further test the satellite separations should be due to oun displacements in the sequences. These separations are 29·21 and 45·19, taking the latter because 21224 is a good measure (B. and W. agree) and the observed $\nu_2 = 19{\cdot}71$ agrees so closely with the Δ_2 value and the ν_1 is subject to the very common triplet modification. These separations require displacements in the sequence mantissæ of $851 - {\cdot}040\xi + 29 d\nu$,* and $1313 - {\cdot}060\xi + 29 d\nu$. Now $59\delta = 853{\cdot}7 \pm 1{\cdot}2$, $90\frac{3}{4}\delta = 1313{\cdot}1 \pm 1{\cdot}8$. This is sufficient to give the satellite multiples, but as the oun is so small, $\delta_1 = 3{\cdot}62$, the close agreement cannot serve as evidence one way or the other as to the satellite nature of the doublet

* WATSON'S value of the separation 29·30 makes this 2·6 larger = 853·6.

and triplet lines in question. The points against the allocation of these lines to the normal D group are, (1) the satellite separations are not in the usual ratio 5 : 3 (they are in fact close to the ratio 3 : 5), (2) there is no appearance of satellite lines corresponding to the parallel F lines noted above, and (3) the limit 53081 is somewhat larger than we should expect from the march of the limits in the other gases.

Taking it, however, as the limit, it is possible with the given value of Δ_1 to calculate the link *e*. The result is $197+4d\nu_1$. The occurrency curve as found from B.'s and W.'s observations is given in Plate 2, fig. 6. The dotted line is the result when the numerous rough measures by L.D. are included. As is seen the maxima occur at 195·6 and 198, the calculated, at a minimum. The peaks look as if analogous to corresponding peaks found in other elements, but the analogy is doubtful. In the other elements these (much larger ones) are produced by the prevalence of displacements by a few ouns, and by Δ_1 operating on $(x\delta)$ S(∞). Such changes here would be very small and the corresponding effects are really shown by the flattened tops of the peaks. The peaks themselves are due in all probability to another cause to be considered shortly (p. 458).

In the region in which the S triplets should be expected are found (all interferential measures)

(6) 28787·86 **49·81** (5) 28837·67 **18·61** (4) 28856·28

with intensities in the proper order, although the ν_1, ν_2 are slightly different from those obtained in the D set. The interferential measures of MEGGERS and of B.M.M. differ considerably (·008) and are not so reliable therefore as usual. BALY and WATSON agree in giving the separations as 49·72, 18·64. The third line, however, has already appeared in the quite definite relation 1429+417 ahead of F(4). It cannot be S (2), but the latter may be a weak line close to it.

There is also another doublet set

(10) 27818·89		**47·75**	(1) 27866·64 ⎫
		49·93	(1) 68·82 ⎭
(8)	·73	**48·07**	(1) 6·80

The suggested S_2 line is abnormally weak. There are, however, here a number of other weak lines which have the appearance of displaced debris. If the first set form a S group, the second would belong to a group with smaller limit, which is also indicated by the smaller triplet separations. The limit 53081·24 was obtained from apparently the only stable D set, whilst the first set are apparently the only stable strong S group. It is natural therefore to take its limit as the same. If so, the limit for the second, being 969·03 (or ·35 $d\nu$) less, is 52112·20. The two mantissæ are $437420-13{\cdot}54\xi$ and $450723-13{\cdot}92\xi+13{\cdot}92d\nu$. Their difference is

$$13303-{\cdot}38\xi+13{\cdot}92d\nu = 20\,(665{\cdot}15-{\cdot}019\xi+{\cdot}7d\nu) = 20\Delta_1.$$

Furthermore the observed separation 47·75 requires, on this new limit, a displacement $664+13{\cdot}9\,d\nu_1$. With $d\nu_1 = {\cdot}11$ we have Δ_1 the same as before, *i.e.*, the 47·86 is the proper ν_1 separation of the new limit. The ν_2 proper to this limit should be 19·15 giving $S_3 = 27885{\cdot}90$. This is not seen, but the *u*-linked line is possibly given by L.D.'s line $27995{\cdot}5 \pm 4 - 106{\cdot}8 = 27988{\cdot}7 \pm 4$. The actual S_2 line appears split up into the additional displaced lines, each (1) 27858·26, (1) 27868·82, (1) 27873·63 from observed S_2. As a parallel *u*-link to the first we find also L.D., $27964{\cdot}2 \pm 4 - 106{\cdot}8 = 27857{\cdot}4 \pm 4$ for actual 58·26. The oun is too small to decide whether the displacements are produced in the limit or the sequent.

The limit for this new set, 52112·20, is more in step with the progression of the other gases, viz. :—

Ne.	A.	Kr.	X.	RaEm.
52112	51731	51651	51025	50403

It is possible to get an estimate of the *u.v* links although there is no means apparent at present of getting the exact value of $s(1)$. Either of our S groups gives the same value for $s(2)$, viz., $53081{\cdot}24-28787{\cdot}86$ or $52112{\cdot}20-27818{\cdot}89 = 24293{\cdot}3$. The denominator of this is 2·1248, so that the $\mu+\alpha/2$ of $s(2)$ is ·1248. We can get an estimate of the value of $s(1)$ from the law that $(\alpha+\frac{1}{2}\Delta_1)/(\mu+\frac{1}{2}\Delta_1)$ is about ·2 in the other periodic groups. The corresponding values in this group are A, ·189 ; Kr, ·198 ; X, ·222. If the ratio ·18 is taken for Ne $\mu+\alpha$ comes to about ·1139 with $s(1)$ about 89000. These are only rough estimates, but the values of *u.v* will only alter slowly with considerable changes in $s(1)$. The values calculated from this $89000+\xi$ with $\Delta_1 = 666$ are

$$u = 106{\cdot}78 + {\cdot}0017\xi, \qquad v = 106{\cdot}86 + {\cdot}0018\xi.$$

This shows that although the value of $s(1)$ may be extremely rough, those of *u.v* may be relied on within a few decimals.

One is inclined to regard the weak S set as that which is analogous to those determined in the other gases, and which certainly belong to the blue spectrum, and that in Neon, which shows only one spectrum, it is composite, and the blue not strongly developed. The question naturally arises whether there is any evidence of an unstable or weak D set with the same limits, *i.e.*, about 969 behind the former D. There is a set in this neighbourhood

(0) 20173·82 **42·86** (2) 20216·68

of the right order of inverted intensities and a small ν_1, possibly a case of the common D triplet modification. With the limit 52112·20 the mantissa of 20173 is 853094, which is 373 greater than that of the old *d*. This is within error limits of $26\delta = 376$ This may be explained by the supposition that the *d* sequence is not affected by the limit and that the present pair run parallel to one 26δ above the old D_{13}. If this existed it should be 21142·64, 21191·98, 21211·69. There are lines for the last two but none for the first. They are

[21142·74] **[49·24]** (0) 21190·61 **20·10** (6) 21210·71.

There is thus considerable evidence for the existence of parallel S groups as well as of parallel D ones. Each set has its corresponding ν_1, ν_2 separations, but with the same oun multiples for Δ_1, Δ_2. We should consequently expect to find the presence of corresponding $a \ldots e$ links. With the S limit 52112 the e link is found to be 191·49. Now the occurrency curve for e gives a maximum between 195·6 and 196 pointing to a S limit as basis about 650 less than 53081 or, say, 52430. This makes $e = 195·63$, $\nu_1 = 48·9$. Do we find evidence for S and D sets about this amount less in wave-number than the old ? For the S we are landed in a region which forms a gap in B.'s, or W.'s observations but which contains a number of lines by L.D. Amongst these we find

	S_1.		S_2.
1429, **195·7**, **105·9** }	27964±4	**47**	28011±4
	213		
	28177±4	**40**	28217±4

The suggested S_1, S_2 fall into the proper positions, the link 1429 is one of WATSON's constants, 195·7 is the e link proper to the limit, 105·9 is u or v, and 213 is $2u$ or $2v$.

For the D sets we find amongst W.'s lines

(0) 20551·35	**49·62**	(2) 20600·97
157·51		
(3) 20708·86		

The 20551 is 601 below $D_{13} = 21156$, which when the variability of the satellites is considered may be taken as the analogue of 610 for the S set. If it is corrected* by $dn = ·7$ $(d\lambda = -·17)$ the ν_1 becomes the correct value 48·9, and it is then also separated from 20708 as a D_{11} line by 156·8, which at once suggests the origin of the 156 parallel F and **F** sets previously brought to light. If this relation be real, the old $F(\infty)$ 31851 is a d satellite sequent, belonging to 20551·35 or ...2·0 and the limit is $20552·0 + 31851·13 = 52403·13$. Not only is the 156 separation found, but there are a large number of lines in the neighbourhood which give, possibly within the error limits, the other separations indicated by the traces of parallel F series adduced at the beginning. Also as indicating a D region we find large repetitions of e and b links

* Not necessarily error, probably the usual D displacement on sequent.

characteristic of D linkages. As referring to the F series we find, starting from 20551—which corresponds to 31851—

				(3) 20556·84		
			44·13			
(0) 20551·35	**49·62**	(2) 20600·97				
		(0) ... 2·02		**85·81**		
114·84				(0) 20642·61		
(5) 20666·19						
157·51						
D_{11} (3) 20708·86						
194·42						
(2) 20745·77						
232·02						
(3) 20783·37						
249·22						
(1) 20800·57						
	269·39			**312·66**		
		[20820·74]	[**48·78**]	(2) 20869·52		
					3·04	
		275·00		(0) ... 72·56		**5·61**
					2·57	
		(1) 20826·35	**48·78**	(4) ... 75·13		

There is also (1) 20542·57 at 166·29 behind the D_{11} line 20708·86 which seems related to Rossi's series referred to later.

The application of the D qualification test is unfortunately here nugatory on account of the smallness of the oun. It is, however, striking that so many of the F separations put themselves in evidence. This agreement adds considerable weight to the allocation. The evidence is also practically convincing when the Watson separations are brought into evidence as is done a few paragraphs below. Support is also given in the doublet system shown in connection with 20826, which show the same small displacements as are exhibited in the first of the parallel F sets adduced above. In this case to 20869 as a D_2 would correspond a [20820] as a D_1 269·39 ahead of 20551, whereas the F is 312·73 behind the corresponding F_1 line (17081). If the two sets correspond, *i.e.*, the d sequent and the $F(\infty)$ for this set are the same, the D group must belong not to 20551 but to a parallel D group in which the limit is $312·73-269·39 = 43·34$ behind the normal $D(\infty)$. Now this is the separation of 20556 behind 20600. In fact the separation of 20556 and 20869 is 312·66. This further indicates that 20556 is a D_2 line in the position shown in the above table, in which the corresponding D_1 line is too weak to have been seen. This also gives another F separation—85—with 20642. The change 44·13 on the D limit requires a displacement on the mantissa of 611 and $42\frac{1}{4}\delta = 611·3$, which is exact. The ν_1 corresponding to this new limit, with $\Delta_1 = 666$, is 48·30. The difference between this and the observed 48·78 is just within our assumed possible errors on two lines.

Again, the double line 20600·97, 02·02 apparently reproduces itself in the F group, as MEISSNER says 17081 shows also a weak component. It is also in evidence in another relation as we shall see immediately.

The existence of parallel F series with WATSON'S separations and also 1932 has already been pointed out, viz.,

• **1932** F(1) **1429** • **422** •

If these depend like the others on D series, the same separations should be found in the reverse order. They do not appear with D_1, but they are seen with the stronger line $D_2 = 20600$, with which also the linkage relation enters. Thus

				1428·68	(2) 20600·97	**1925 ± 3**	*22526*
1428·46	(8) 18753	**419·13**	(1) 19172			**1429·45**	(4) 22030
				1429·73	(0) 20602·02	**1428·40**	

in which there is also a mesh between 19172 and 22030 and a link back from 18753. There may also be a forward **1932** linked line if L.D. has $d\lambda = -1·1$. It is striking that a mesh is repeated in the F(1) set. Thus

	1429·38	18510·84	**422·50**	
17081·46				18933·34
	1430·24	18511·70	**421·64**	

The separation of the 20600 lines is practically repeated in the 18510.

The parallel D set, with $D_2 = 20556$, also shows traces of the separations, with the mid lines not seen, in analogy with the weak mid one just considered. Thus

(7) 18709 **417·86** [19127] **1429·42** (3) 20556·84

The absence of the corresponding lines connected to 20551 may be due to the scattering of the lines by displacement. The wanting lines should be [19122·07], [18704·6], the former corresponding to the weak 19172 of the second D group. Now we find strong lines corresponding to both these at the same separations ahead and suitable for the same displacements, viz.,

[19122·07]	(1) 19172·29
20·79	**20·27**
(5) 19142·86	(4) 19192·56

These numerous F and D relations render it certain that the sets of lines adduced belong respectively to sets of F series and the D series. Moreover, it suggests that the source of the 1429, 417 separations is the d sequent or $F(\infty) = 31851{\cdot}13$.

A possible supposition is that their source should be in the $S(\infty)$ limit. If so we should expect it to appear strongly in the S lines, and so in the $S_1(2)$ lines considered above. As a fact, however, there is no sign of such in either of the S groups adduced, except a very dubious one 1425 between two L.D. lines each of which has an ambiguity ± 4. It takes place between $S_1 = 27964$ and 26539 backwards, so that if its source were here it would be a positive displacement on $S(\infty)$ or a negative one on $s(2)$, both unusual. The strongest argument for its source being in the 31851 is that the separations in question show themselves in *all orders* of $F(m)$—in other words, occur in the limit $F(\infty)$.

The Value of the Oun.—It has already been found that the value of the oun calculated from the chemist's atomic weight is $14{\cdot}47 \pm {\cdot}01$ and that the oun multiple for Δ_2 is $18\frac{1}{2}$ or $\Delta_2 = 267{\cdot}70 \pm {\cdot}02$. This is too small—or the inexactness too large—to obtain a more accurate value as in the other cases directly from the F or D mantissæ. It is, however, possible to arrive at an extremely accurate estimate by proceeding step by step with successive approximations, and for this purpose the F separations are clearly at disposal. The wave-lengths of many of the F (1) lines are very accurately known (B.M.M. will be used), they are so large that the dn are small multiples of $d\lambda$, and being of order $m = 1$, an oun displacement will produce a comparatively large change in n. In spite, therefore, of the smallness of the oun it is possible to get some definite information. The reliability of the information will depend on two assumptions—

(1) That the lines employed are F lines parallel to the series $F(1) = 17081$.
(2) That no displacements occur in the f sequents themselves.

If the assumption (2) is not satisfied the series in question will not show constant separations from the corresponding $F_1(m)$ lines, but will converge or diverge with increasing order. The lines we shall make use of have been measured probably up to a few thousandths of an Ångstrom, and the accuracy is greater than one in the seventh digit in the value of n. Moreover, in calculating with seven-figure logarithms, in which also we have to do with differences between two numbers, errors amounting to unity or more are liable to enter. Consequently where these very accurate numbers occur nine-figure logarithms have been used. As the wave-lengths are given in I.A. the calculations have been made on that basis. The limit $31851{\cdot}1300$ R $= 31852{\cdot}1816$ I. $N = 109678{\cdot}6$. Put $\Delta_2 = 267{\cdot}70 + x$, therefore $\delta = 14{\cdot}4703 + {\cdot}054x$ where at present x lies between $\pm{\cdot}2$. It should be noted that in the d sequences, the satellite displacements are not in general multiples of δ, but of $\delta_1 = \frac{1}{4}\delta$. The correct value of a wave-length will be taken as the observer's value $-{\cdot}001 \times p$, so that $dn = + \ldots$.

Amongst the sets of F series given (p. 452) two, in addition to F (1) itself have been measured to the required degree of accuracy. They are those showing separations of 85 and 265. Under these conditions

$$\text{Mantissa of } 31852\cdot1816+\xi = 855630\cdot30-29\cdot130\xi$$

$$\text{Wave-numbers of } 5852\cdot4870 = 17082\cdot0220-\cdot0029p_1$$

$$5881\cdot8958 = 16996\cdot6140-\cdot0029p_2 \qquad \mathbf{85\cdot4080-\cdot0029\,(p_1-p_2)}$$

$$5944\cdot8344 = 16816\cdot6702-\cdot0029p_3 \qquad \mathbf{265\cdot3518-\cdot0029\,(p_1-p_3)}$$

(1) *Separation* = 85·41.—This is the same within error limits as occurs in the D series, but the corresponding F series shows ν increasing to 87·57 W or 87·41 B at $m = 5$, which means additional displacements either in F (∞) or the sequences. As the 85 agrees in both the F (1) and D series this will not happen in the F (∞), and the separation 85·41 will be due only to the actual separation in 31851. The limit of the F series in question is therefore 31852·4170−85·4080 = 31765·8336+ξ.

$$\text{Its mantissa} = 858133\cdot15-\cdot085\,(p_1-p_2)-29\cdot247\xi$$

$$\text{Difference from } F_1 = 2492\cdot85\ldots$$

Now

$$9\Delta_2+5\tfrac{3}{4}\,\delta = 2492\cdot504+9\cdot311x, \qquad x < \cdot2$$

Hence

$$9\cdot311x = \cdot35-\cdot085\,(p_1-p_2)-\cdot117\xi$$

$$x = \cdot038-\cdot009\,(p_1-p_2)-\cdot0126\xi$$

The important point to notice is that with our preliminary limit of uncertainty ($x < \cdot2$), the oun multiple cannot be any other than that used, so that the second approximation is quite definite. It has already been seen in the discussion of the F_1 series that ξ is probably within ±·33 also p_1-p_2 will not numerically be greater than 4. Hence $x = \cdot038\pm036\pm0042 = \cdot038\pm\cdot04$

$$\Delta_2 = 267\cdot738\pm\cdot04$$

$$\delta = 14\cdot4705\pm\cdot0009$$

(2) *Separation* = 265·3518.—Limit = 31586·8298+ξ+·0029 (p_1-p_3)

$$\text{Mantissa} = 863408\cdot31-\cdot085\,(p_1-p_3)-29\cdot498\xi$$

$$\text{Difference from } F_1 = 7778\cdot01-\cdot085\,(p_1-p_3)-\cdot368\xi$$

Now

$$29\Delta_2+\delta = 7777\cdot77+29\cdot05x$$

therefore

$$29\cdot05x = \cdot23-\cdot085\,(p_1-p_3)-\cdot368\xi$$

$$x = \cdot008-\cdot0029\,(p_1-p_3)-\cdot0126\xi = \cdot008\pm\cdot016$$

$$\Delta_2 = 267\cdot708\pm016$$

In comparing these values it must be remembered that ξ and p_1 enter in both. Equating the two, ξ disappears and

$$\cdot030-\cdot006p_1+\cdot009p_2-\cdot003p_3 = 0$$

This is easily satisfied by possible observation errors—*e.g.*, $p_1 = -p_2 = p_3 = 1\cdot4$ say,

$$\Delta_2 = 267\cdot713-\cdot0126\xi\pm\cdot01$$

(3) WATSON's *Separations.*—When the strong lines giving these separations are taken, the exactness of the equality of the separations obtained from them is most remarkable. Using the interferentially measured lines by PRIEST, MEISSNER, and MEGGERS, with 9-figure logarithms the following values are found in I.A. for the means

1429·4292	8	·0065	·0048
417·4533	7	·0120	·0064
1070·075	1		

The last enters as a component of 1429, viz., 1429 = 1070+359.

The second column gives the number of the observations used, the third the maximum deviation of a single observation from the mean, and the last the root mean square of all the deviations. Using these the mantissæ of the following numbers have to be found with $\xi+d\nu$ added—

31852·1816 **1429·4290** 33281·6106 **417·453** 33700·0636, **1070·075** 32922·2530

They are

$$855630\cdot30-29\cdot130\xi$$
$$\mathbf{40286\cdot52-1\cdot857\xi+27d\nu_1}$$
$$815343\cdot78-27\cdot273\,(\xi+d\nu_1)$$
$$\mathbf{11278\cdot99-\cdot505\xi+27d\nu_2}$$
$$804064\cdot79-26\cdot768\,(\xi+d\nu_1+d\nu_2)$$

$$825224\cdot33-27\cdot721\,(\xi+d\nu_3) \qquad \mathbf{9980\cdot55-\cdot448\xi-27\,(d\nu_3-d\nu_1)}$$

In this particular case, $F_1 = 17082$, and F_1+1429 have both been observed interferentially and the separation is $17082\cdot0240-18511\cdot4499 = 1429\cdot4259$, or $d\nu_1 = -\cdot0031$. The 417 separation is altered by some displacement to $422\cdot52\pm\cdot17$ and is therefore not directly applicable. The calculations have been carried out on the basis of the values obtained on the averages. It must be remembered, however, that when displacements enter in a sequent they frequently occur on values of the sequent which have already received a small displacement, in which case the separations themselves receive small changes. Too much weight must therefore not be given to the 417 case here, in which its actual value for the particular set is not obtainable. The $d\nu_2$, however, is certainly very small.

The discussion of the two F separations has given $\Delta_2 = 267{\cdot}713 - {\cdot}0126\xi \pm {\cdot}01$, $\delta = 14{\cdot}4710 - {\cdot}00068\xi \pm {\cdot}0005$. Let x denote the correction required on this value of Δ_2. Then

$$36\Delta_2 + 16\tfrac{3}{4}\delta = 9880{\cdot}057 - {\cdot}4653\xi + 36{\cdot}90x$$
$$42\Delta_2 + 2\tfrac{1}{2}\delta = 11280{\cdot}123 - {\cdot}5306\xi + 42{\cdot}13x$$
$$150\Delta_2 + 9\delta = 40287{\cdot}19 - 1{\cdot}896\xi + 150{\cdot}48x$$

Supposing that the true multiples of the oun are y greater, and putting $d\nu_1 = -{\cdot}0031 + d\nu_1$

$$36{\cdot}90x - {\cdot}017\xi + 27\,(d\nu_3 - d\nu_1) - {\cdot}40 + 3{\cdot}62y_3 = 0$$
$$42{\cdot}13x - {\cdot}025\xi - 27d\nu_2 + 1{\cdot}13 + 3{\cdot}62y_2 = 0$$
$$150{\cdot}48x - {\cdot}040\xi - 27d\nu_1 + {\cdot}75 + 3{\cdot}62y_1 = 0$$

or

$$x = {\cdot}010 + {\cdot}00046\xi - {\cdot}73d\nu_3 - {\cdot}097y_3$$
$$x = -{\cdot}027 + {\cdot}00057\xi + {\cdot}64d\nu_2 - {\cdot}086y_2$$
$$x = -{\cdot}0050 + {\cdot}00026\xi + {\cdot}18d\nu_1 - {\cdot}024y_1$$

In these ξ cannot be more than a few units, $d\nu < {\cdot}02$ and $x < {\cdot}01$. This can only happen if all the $y = 0$. Thus again there is the very important fact that the oun multiples are quite definite and are those used in the actual calculations. ξ is not large enough to affect the limits of accuracy in x. The separation 1070 is not so well determined as the others and $d\nu_3$ may well be $> {\cdot}01$. Thus the first and third can easily give the same values of x, but the second would require $d\nu_2$ of the order ·03, inadmissible if the ν_2 were accurately determined. But as a fact the average ν_2 as we have seen does not enter in the line here considered and it may be so large as to alter the multiple. The second may therefore be considered as not at disposal, and the third then gives very close limits, viz., with $d\nu_1 \not> {\cdot}01$

$$\Delta_2 = 267{\cdot}708 - {\cdot}0124\xi \pm {\cdot}002$$
$$\delta = 14{\cdot}4708 - {\cdot}0007\xi \pm {\cdot}0001$$

the same value, though with closer limits of accuracy, as was obtained from the 265 separation. With maximum $\xi = {\cdot}33$, $\delta = 14{\cdot}4708 + {\cdot}0003$.

But further, in addition to WATSON's separations, we have found affixed to the F series, another $= -1932$, and this must be tested. The linked line is given by B.M.M. as 6598·953 I.A. Still using 9-figure logarithims, the wave-number is 15149·7338 giving the separation 1932·2902, and corresponding to a limit 31852·1816 $-1932{\cdot}2902 = 29919{\cdot}8914 + \xi - d\nu$. The mantissa of this is $914613{\cdot}17 - 31{\cdot}997\,(\xi - d\nu)$. The displacement on 31852 is therefore $58982{\cdot}87 - 2{\cdot}867\xi + 32d\nu$. With the above values of Δ_2, δ, $220\Delta_2 + 6\delta = 58982{\cdot}58 - 2{\cdot}728\xi \pm {\cdot}44$. This again is an exact agreement.

The foregoing does not give Δ_2 with the desired definiteness unless the value of ξ is determined. The reason is that it has been based on displacements on the same limit.

An independent datum is necessary to fix the value of ξ, which at present we know cannot be greater than a few units. In the other gases this is obtainable by the conditions that the mantissa of $f(1)$ is a multiple of Δ_2 and that of F (∞) of δ_1. The value of F (1) is known with great exactness, probably to within less than ·001A. PRIEST gives $\lambda = 5852·4862$, MEISSNER ·4875, B.M.M. ·488. They all agree within ·001 of ·487 so that the wave-number of F (1) is $17082·0242+·0029p$ with p within ± 1. But here we have to answer the question whether our first F line 17082 is really the first of the series. If this F series—a very marked and definite one—belongs to the 1864XF type it has its first line for $m = 1$ in the ultra-red (calculated $n = -8168·62\lambda = 12242$) and far beyond the reach of any sounders. This uncertainty will therefore have to be kept in mind. The mantissa of 17082 must in any case be a multiple of δ_1, and may be of Δ_2 (if it is the first F line). In addition the D set 20551 may belong to that D satellite set with M (Δ_2), or since the ν_1 separation is modified, differ from a multiple of Δ_2 by a few ouns only. We have then

$$\text{Mantissa F}(\infty) = 855630·30-29·130\xi = \text{M}(\delta_1) \text{ and possibly M}(\Delta_2)$$

$$\text{,,} \quad f(1) = 725012·36-92·257\xi-·2675p = \text{M}(\delta_1) \text{ and possibly M}(\Delta_2).$$

The term in p will not affect our immediate purpose and may be omitted. There is evidence for $\xi < ·33$. Also $\Delta_2 = 267·708-0124\xi+x$ with $x < ·002$. We find

$$2708\Delta_2 = 724953·26-33·58\xi+2708x, \qquad 2708x < 5·4$$

$$3196\Delta_2 = 855594·77-39·63\xi+3196x, \qquad 3196x < 6·4$$

Hence

$$\text{Mantissa of F}(\infty) = 3196\Delta_2+35·5+10·5\xi-3196x$$

$$\text{,,} \quad f = 2708\Delta_2+59·10-58·68\xi-2708x.$$

A first definite result is that 20551 is not a D line of the M (Δ_2) type. Its mantissa must differ from such by at least 7 or 8 ouns. The coefficient of ξ is so large that the actual oun multiple cannot be uniquely decided. Further f cannot follow the Δ_2 multiple law unless ξ be of the order 1, or three times our estimated limit of variation. We cannot say that it is impossible. If however this F line is the first of the series then ξ must be of this magnitude. It is seen that no further definite and certain information can be obtained. It will however be of some interest to follow out the assumption that its mantissa is a M (Δ_2). In this case put $\xi = 1+\xi$ where now ξ is small, and

$$·42-2708x-58·68\xi-·2675p = 0$$

The uncertainties in ξ are now $·42\pm 5·4-59\xi\pm ·26 = 0$, or $\xi < ·1$.

$$\Delta_2 = 267·6957-·0124\xi+x.$$

In conclusion some reference is necessary to the series due to ROSSI.* These

* *Loc. cit.*

consist of two doublet series each with separation of order 167 and a third singlet series. The two sets of doublet series appear to converge to the same limits, in the same way as the satellites of a D series. The first lines given by him are of the order $m = 3$, but amongst the B.M.M. as also the MEISSNER lines those for $m = 2$ are also seen. Further, there appear close strong companions to the second lines of doublets in the first series, and in this they recall the behaviour of the F lines in the alkaline earths* and so suggest that they are really lines of the F type, but belonging to a different f sequence from that discussed above. If so the limit must behave as a displaced value from the 31852. The sets are, in ROWLAND units:—

m.					
			165·44	(8) 11932·92	
					1·73
2	†	(4) 11767·48	**167·17**	(0) 4·65	
		107·63			
	†	(2) 11875·11	**168·29**	(4) 12043·40	
				(8) 17342·68	
					1·06
3	†	(5) 17176·58	**167·16**	(1) 3·74	
		46·39			
	†	(5) 17222·97	**168·21**	(5) 17391·18	
	†	(6) 19677·64	**166·58**	(6) 19844·22	
4		**23·92**			
	†	(4) 19701·56	**167·86**	(5) 19869·42	
	·04	(5) 21033·94	**166·87**	(5) 21200·81	
5		**14·08**			
	−·02	(4) 21048·02	**167·59**	(3) 21215·61	
	·04	(2) 21850·62	**167·08**	(3) 22017·70	
6		**10·17**			
	−·25	(1) 21860·79	**169·63**	(4) 22030·42	
	·06	(3) 22380·18	**167·18**	(2) 22547·36	
7		**9·91**			
	−·01	(0) 22386·29‡			
8	·07	(0) 22742·78	**167·31**	(0) 22910·09	
9	·20	[23168−167]		23168 (L.D.)	

* [III., p. 383, *seq.*]

† Lines used in calculating formulæ.

‡ This line was not included by ROSSI.

The formulæ calculated from the first three lines of each series give

$$n = 24104{\cdot}67 - \mathrm{N}/\{m + {\cdot}973110 + {\cdot}016932/m\}^2$$
$$n = 24104{\cdot}48 - \mathrm{N}/\{m + {\cdot}987215 + {\cdot}014948/m\}^2$$

The O−C values for these are given in the above list before the wave-numbers. They are larger than we should expect the observation errors to be. If, however, the limit be reduced by ·40 and μ, α calculated from the first two, the O−C for $m = 4$ is −·04 and zero for all the others, $m = 5 \ldots 8$. For $m = 9$, using L.D. $23168 \pm 3 - 167$ as the observed for the first of the pair the O−C is ·2. The agreement for all is therefore exceedingly close except for $m = 4$. The calculated wave-number for this is 19677·50 which makes the separation 166·72 and more in step with the others. The uncertainty in the limit 24104·67 must therefore be very small.

The doublet separations show a tendency to converge with increasing order, but this is clearly due to the fact that the constant separation must be taken between the strong first line and the weak second. After $m = 3$ the weak is not seen and the observed separation is not the true one but that between the first and the second strong one. The separation in the second series is somewhat larger than in the first, that for the first being 167·17 and for the second $168{\cdot}25 \pm 04$ (mean for $m = 2, 3$). These require limit mantissæ changes of 7354 in the first where $308\frac{1}{4}\delta = 7354{\cdot}1$, and of $47{\cdot}79 \pm 1{\cdot}76$ extra for the other where $3\delta = 46{\cdot}41$. The separation 1·73 between the four satellites in the first doublet must be due to displacement in the sequent. It requires a mantissa change of 209 and $14\frac{1}{2}\delta = 209{\cdot}8$.

These considerations point strongly to the conclusion that the series are of the F type. Fortunately, owing to the fact that the first line of the first series has been very accurately measured by B.M.M., as $\lambda = 8495{\cdot}380$ I.A., it is possible to test if the mantissa of $f(2)$ is $\mathrm{M}(\Delta_2)$. Taking the observation error as $-{\cdot}001p$, the wave-number is $11767{\cdot}8680 + {\cdot}0014p$. Its mantissa with limit $24105{\cdot}0731 + \xi$ (the I measure of 4·2700 R.) is

$$981622{\cdot}48 - 120{\cdot}87\xi + {\cdot}17p = 3667\,\{267{\cdot}6910 - {\cdot}03296\xi + {\cdot}00005p\} = 3667\Delta_2$$

within the uncertainty of ξ.

The first line of the second series has not been measured so exactly. Its $\lambda = 8418{\cdot}38 - {\cdot}02p$; $n = 11875{\cdot}50 + {\cdot}028p$; mantissa $= 13092 - 1{\cdot}56\xi + 3{\cdot}52p$ larger than the other. Now $49\Delta_2 - 7\delta_1 = 13091{\cdot}8$. Hence if the two limits are the same 24104·89 (*i.e.*, put $\xi = -{\cdot}19$), the mantissa is larger by $49\Delta_2 - 7\delta_1 + {\cdot}51 + 3{\cdot}42p$. It is satisfied by $p = -{\cdot}14$ or $d\lambda = {\cdot}002$. But it is possible that these F are due to independent groups, *i.e.*, that the sequence of the second series also depends on a whole multiple of Δ_2. This cannot happen unless the two limits are different, which in fact seems to be the general rule. If the limit is displaced by $y\delta_1$, ξ is $3{\cdot}617y/44{\cdot}25 = {\cdot}082y$, and the mantissa difference from $49\Delta_2$ is now

$-7\delta_1 + \cdot 51 + 10\cdot 04y + 3\cdot 42p = 0$ or $y = 2\cdot 4 - \cdot 34p$. In other words $y = 2$, $p = 1$. This makes the limit of the second series less than that of the first by ·17, in agreement with the relative values as actually found by direct calculation from the observed lines. Although therefore the limits suggest equality, there is good evidence that the observed difference, though small, is real, and is due to a displacement of two ouns. The sequence depends on an origin of $3667 + 49 = 3716\Delta_2$.

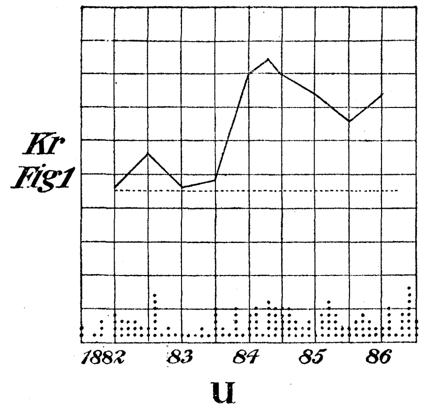
Rossi's third series is (6) 28788·08, (2) 34087·11, (1) 36536·62, (1) 37869·0, the last being observed by himself. The first three give

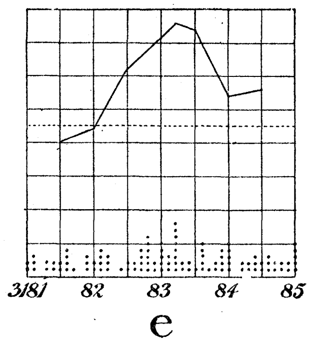
$$n = 40896\cdot 73 - \mathrm{N}/\{m + \cdot 024118 - \cdot 043614/m\}^2$$

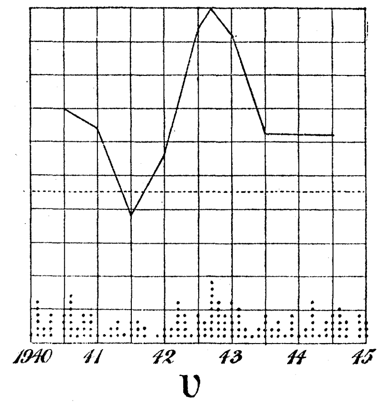
with O−C = −·36 for the last, probably excessive. The first line is the line adopted as one of the S_1 (2) group above, the third is **F** (4). I feel some doubt, therefore, as to these forming a real series, although the sequence has all the appearance of the $s(m)$ type for these gases. The limit does not seem to have any relation to the doublet set. Its denominator is about 1·63.

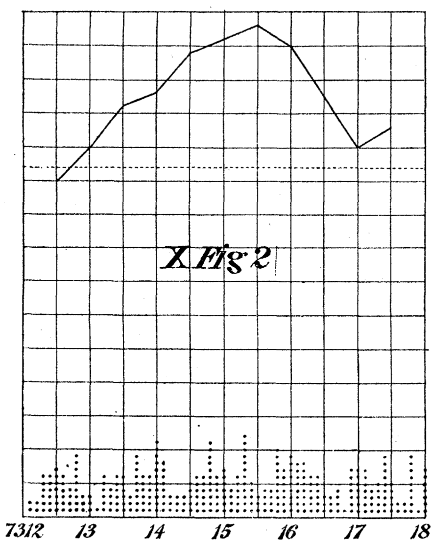
TABLE of Constants.

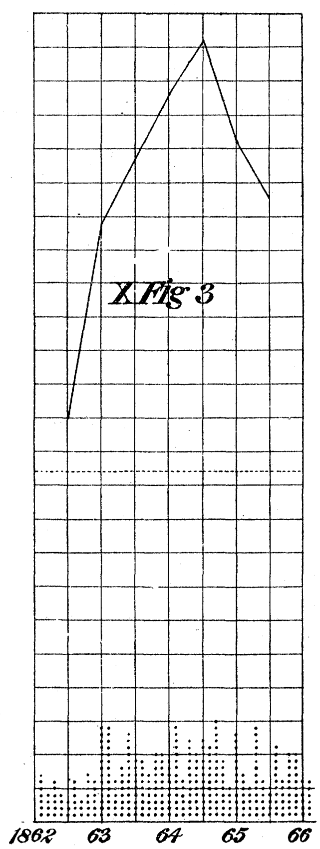
	Ne.	A.	Kr.	X.	RaEm.
ν_1	49·24	179·50	786·45	1777·90	5370·7
ν_2	19·71	75·60	{ 309·20 341·16 }	815·05	2649·53
Δ_1	666	2519	10969	24893	72820
Δ_2	267·708	1057·057	{ 4242·18 4678·80 }	10998·14	32166·44
δ	14·47013	57·9209	249·536	611·0100	1787·024
Δ_1/δ	46	40¾	44	40¾	40¾
Δ_2/δ	18½	18¼	{ 17 18¾ }	18	18
e	191·5	719·71	3183·34	7314·1	23678·4
u	106·78 ?	439·47	1884·03	4133·18	11191·8
v	106·86 ?	442·67	1942·44	4428·00	13680·0
S(∞)	{ 52112 52403 }	51731·05	51651·29	51025·29	50403·00

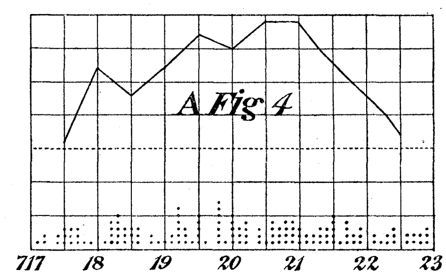

Kr
Fig1
1882 83 84 85 86
u

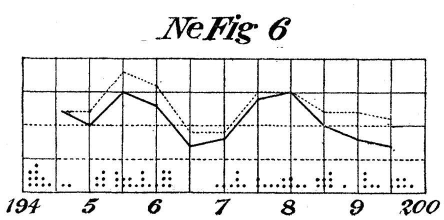

3181 82 83 84 85
e

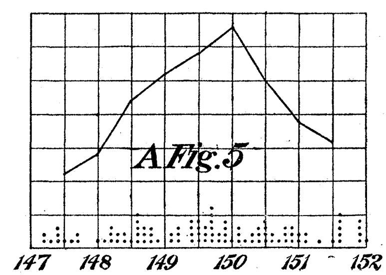

1940 41 42 43 44 45
υ

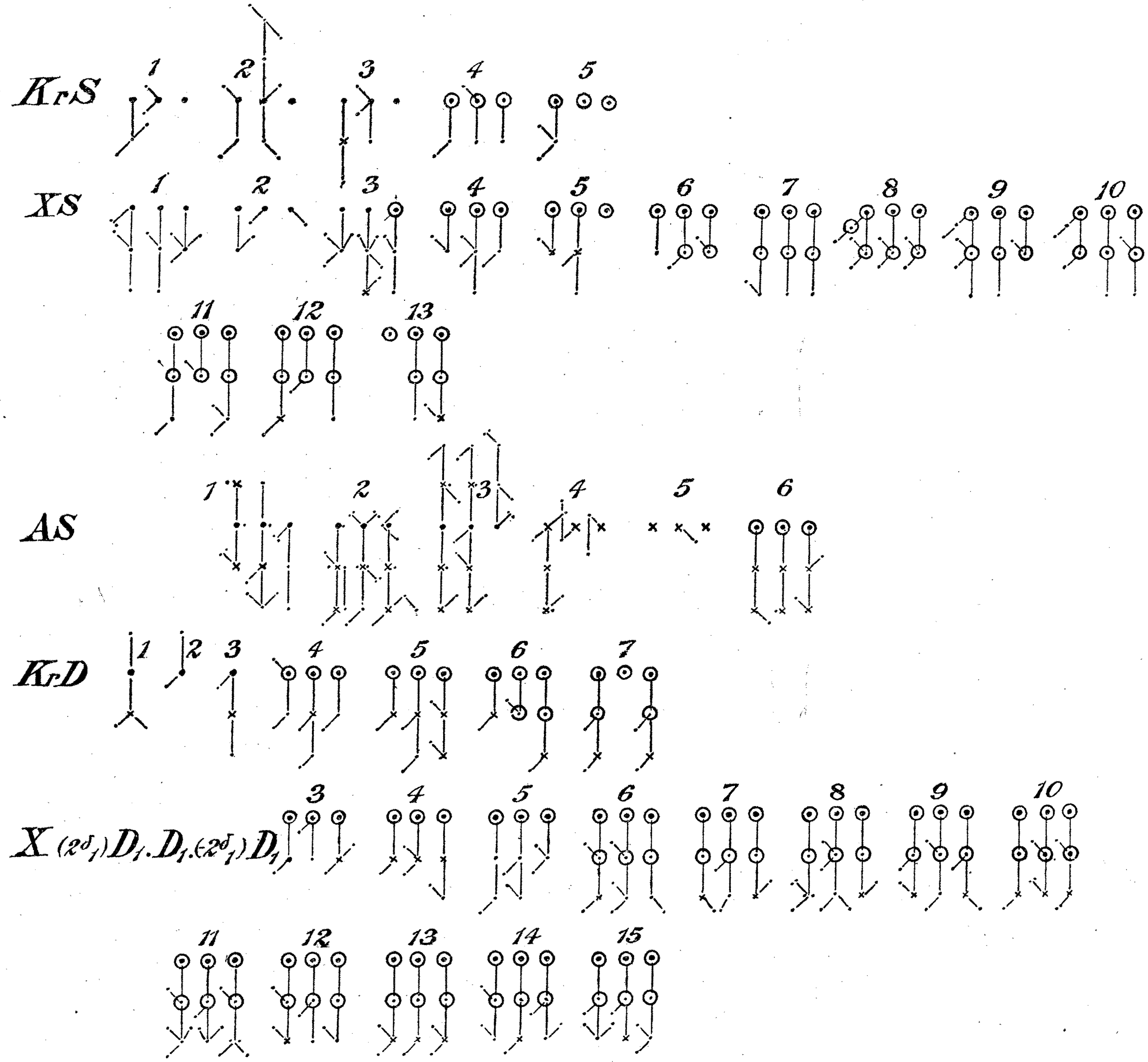

X Fig 2
7312 13 14 15 16 17 18


X Fig 3
1862 63 64 65 66


A Fig 4
717 18 19 20 21 22 23


Ne Fig 6
194 5 6 7 8 9 200


A Fig.5
147 148 149 150 151 152



EXPLANATION OF DIAGRAM.

(1) A dot represents a wave-number; a large dot the wave-numbers being sounded for.

(2) A dot to the side of a line denotes a displaced line.

(3) A circle round a dot denotes that it is in an unobserved region.

(4) A × denotes that the wave-number has not been seen.

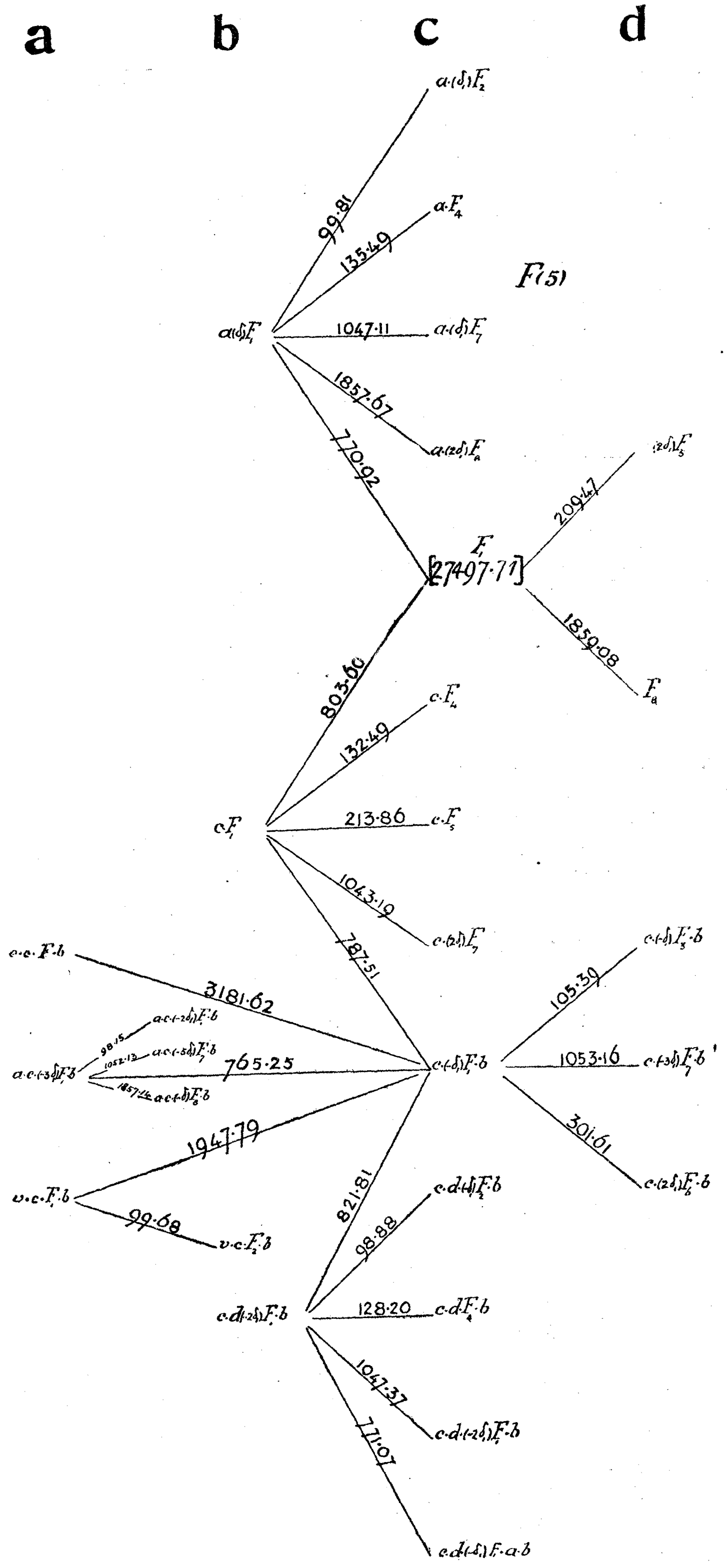
(5) The e links are represented by vertical lines, the u by lines at 45 degrees above the horizontal, the v by lines at 45 degrees below the horizontal.

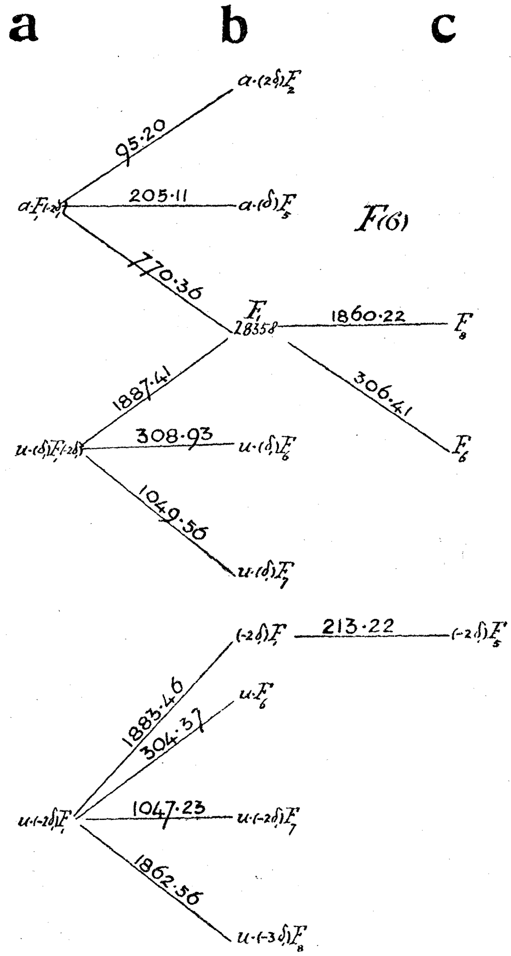
(6) The e link is to be subtracted when drawn down and added when drawn up. The u, v links are to be subtracted when drawn to the left and added when drawn to the right.

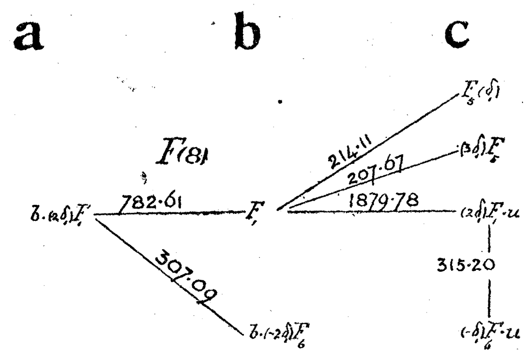
E.g., XS_2 (3) is seen; also the lines linked to it by $-e$, $-e-u$, $-e+u$, $-e+v$; that linked by $-2e$ is not seen, but by $-2e+u$ is.

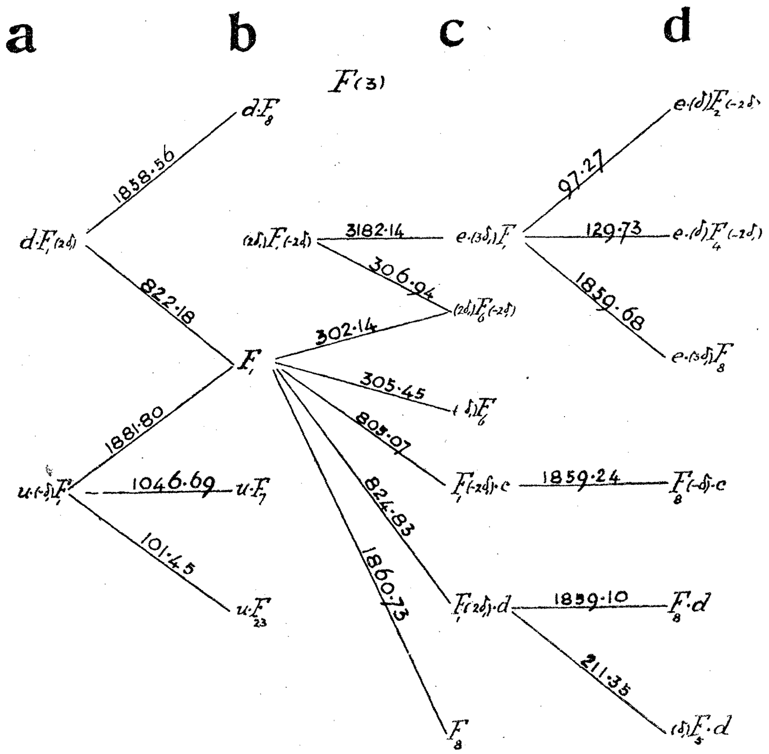
XS_3 (3) is out of the observed region, but the lines linked to it by $-e$, $-e-u$, $-2e$ are in the observed region and have been seen.

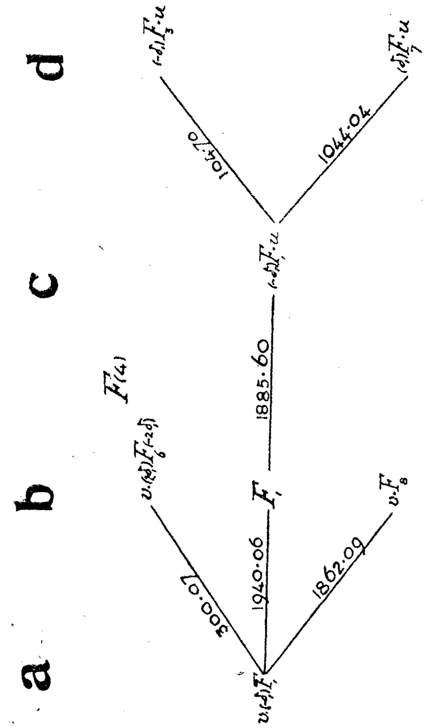
AS contains several examples of displaced lines.

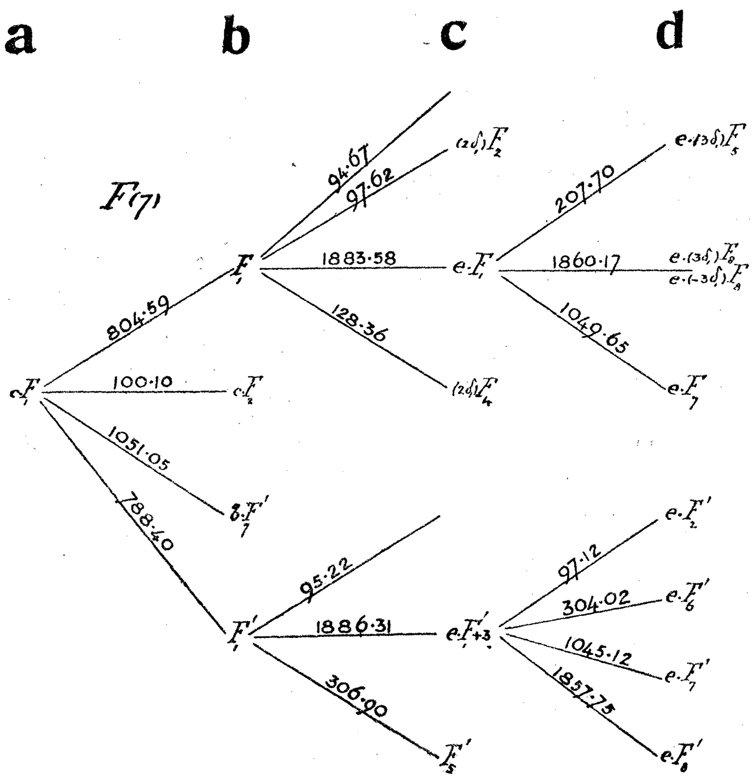

a
b
c
d
F(5)
99·81
135·49
1047·11
1857·67
770·92
[27497·71]
209·47
1859·08
803·60
132·49
213·86
1043·19
787·51
3181·62
765·25
1947·79
99·68
105·39
1053·16
301·61
821·81
98·88
128·20
1047·37
771·07


a
b
c
F(6)
95·20
205·11
770·36
28358
1860·22
306·41
1887·41
308·93
1049·56
213·22
1883·46
304·37
1047·23
1862·56


a
b
c
F(8)
782·61
307·09
214·11
207·67
1879·78
315·20


a b c d
F(3)
1858·56
822·18
3182·14
306·94
302·14
97·27
129·73
1859·68
305·45
805·07
1859·24
824·83
1860·73
1859·10
211·35
1881·80
1046·69
101·45


a
b
c
d
F(4)
1047·0
1044·04
1885·60
300·07
1940·06
1862·09


a b c d
F(7)
94·67
97·62
1883·58
128·36
207·70
1860·17
1049·65
804·59
100·10
1051·05
788·40
95·22
1886·31
306·90
97·12
304·02
1045·12
1857·75